

36 Show that the number axioms become inconsistent when we add the axiom
$$-\infty < y < \infty \Rightarrow x/y \times y = x$$

After trying the question, scroll down to the solution.

Solutions

§	$-\infty < y < \infty \Rightarrow x/y \times y = x$	instantiate
\Rightarrow	$-\infty < 0 < \infty \Rightarrow 1/0 \times 0 = 1$	direction, identity
$=$	$1/0 \times 0 = 1$	symmetry
$=$	$0 \times (1/0) = 1$	multiplication-division
$=$	$(0 \times 1)/0 = 1$	base
$=$	$0/0 = 1$	
	$-\infty < y < \infty \Rightarrow x/y \times y = x$	instantiate
\Rightarrow	$-\infty < 0 < \infty \Rightarrow 2/0 \times 0 = 2$	direction, identity
$=$	$2/0 \times 0 = 2$	symmetry
$=$	$0 \times (2/0) = 2$	multiplication-division
$=$	$(0 \times 2)/0 = 2$	base
$=$	$0/0 = 2$	
	\top	previous two results
$=$	$0/0 = 1 \wedge 0/0 = 2$	symmetry, transitivity
\Rightarrow	$1=2$	transparency
$=$	$(1-1)=(2-1)$	arithmetic
$=$	$0=1$	direction
$=$	$0=1 \wedge -\infty < 0 < 1 < \infty$	specialization
$=$	$0=1 \wedge 0 < 1$	exclusivity (generic)
$=$	\perp	

We have proven \perp so we have inconsistency.