350 (flipping switch) A two-position switch is flipped some number of times. At each time (including initially, before the first flip) there is probability $1/2$ of continuing to flip, and probability $1/2$ of stopping. The probability that the switch ends in its initial state is $2/3$, and the probability that it ends flipped is $1/3$.

(a) Express the final state as a probability distribution.

(b) Equate the distribution with a program describing the flips.

(c) Prove the equation.

After trying the question, scroll down to the solution.

(a) Express the final state as a probability distribution.

§ $(ok + 1)/3$

When there is no state change, ok is 1 ; when there is a state change, ok is 0 . Let the switch position be represented by binary variable b . Then I can rewrite it as

$((b'=b) + 1)/3$

I could further rewrite $b'=b$ as $2\times b'\times b - b' - b + 1$, but I see no point.

(b) Equate the distribution with a program describing the flips.

§ $(ok + 1)/3 \;=\;$ **if** $1/2$ **then** $b{:=}\ \neg b$; $(ok + 1)/3$ **else** ok **fi**

(c) Prove the equation.

§	**if** $1/2$ **then** $b{:=}\ \neg b$; $(ok + 1)/3$ **else** ok **fi**	replace first ok
=	**if** $1/2$ **then** $b{:=}\ \neg b$; $((b'=b) + 1)/3$ **else** ok **fi**	use the Substitution Law
=	**if** $1/2$ **then** $((b'=\neg b) + 1)/3$ **else** ok **fi**	$(b'=\neg b) = \neg ok = 1-ok$
=	**if** $1/2$ **then** $(2 - ok)/3$ **else** ok **fi**	replace **if**
=	$(2 - ok)/3/2 \;+\; ok/2$	arithmetic
=	$(ok + 1)/3$	