306 (nondeterministic assignment) Generalize the assignment notation $x:= e$ to allow the expression e to be a bunch, with the meaning that x is assigned an arbitrary element of the bunch. For example, $x:= nat$ assigns x an arbitrary natural number. Show the standard binary notation for this form of assignment. Show what happens to the Substitution Law.

After trying the question, scroll down to the solution.

§ $x := e \;=\; x' : e \land y' = y \land ...$

$$
\begin{array}{ll}
 & x := e.\ P \\
= & \exists x'', y'', ...\cdot\ (x'' : e \land y'' = y \land ...) \land (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } P) \\
= & \exists x''\cdot\ x'' : e \land (\text{substitute } x'' \text{ for } x \text{ in } P)
\end{array}
$$

but the one-point law does not allow us to get rid of $\exists x''\cdot$. For example, in one variable,

$$
\begin{array}{ll}
 & x := 0, 1.\ x' = x+x \\
= & \exists x''\cdot\ x'' : 0,1 \land (\text{substitute } x'' \text{ for } x \text{ in } x' = x+x) \\
= & \exists x''\cdot\ x'' : 0,1 \land x' = x''+x'' \\
= & x' = 0+0 \lor x' = 1+1 \\
= & x' : 0, 2
\end{array}
$$

but the Substitution Law would give

$$
\begin{array}{ll}
= & x' = (0, 1)+(0, 1) \\
= & x' = 0, 1, 2
\end{array}
$$