292 (coin weights) You are given some coins, all of which have a standard weight except possibly for one of them, which may be lighter or heavier than the standard. You are also given a balance scale, and as many more standard coins as you need. Write a program to determine whether there is a nonstandard coin, and if so which, and whether it is light or heavy, in the minimum number of weighings.

After trying the question, scroll down to the solution.

§ Let c be the number of coins you are given (not including any extra standard coins). The number of possible outcomes is $2 \times c + 1$ because each of the c coins might be light or heavy, or all coins might be standard. Each weighing has 3 possible outcomes: left side heavy, right side heavy, or balance. So w weighings can say 3^w things. So the minimum number of weighings is the smallest w such that $2 \times c + 1 \le 3^w$. Or, the maximum number of coins that can be solved in w weighings is $c = (3^w - 1)/2$. Then in $w+1$ weighings we can solve

$$(3^{w+1} - 1)/2 \;=\; 3 \times (3^w - 1)/2 + 1 \;=\; 3 \times c + 1$$

coins. So one more weighing takes us from c to $3 \times c + 1$ coins. That calculation motivates the following program outline.

(solve for c coins) $\Leftarrow$
    if c=0 **then** all 0 coins are standard
    else if c=1 **then** test the 1 coin against a standard coin
        else if c=2 **then** test each of the 2 coins against a standard coin
            else (solve for *floor* (c/3) supercoins). each supercoin is 3 coins
                if all supercoins are standard
                then (solve for c – *floor* (c/3) coins) 0, 1, or 2 leftovers
                else **if** a specific supercoin was heavy
                    then balance 2 of these 3 coins against each other.
                        if one coin is heavy **then** that's the culprit
                        else the remaining coin is heavy **fi**
                    else some specific supercoin was light, so
                        balance 2 of these 3 coins against each other.
                        if one coin is light **then** that's the culprit
                        else the remaining coin is light **fi fi fi fi fi fi**

That solution requires the use of as many standard coins as the given number c of coins. For example, if there are 9 given coins, then we need to solve for 3 supercoins, and to do that, we need to solve for 1 supersupercoin consisting of 9 coins, and to do that we need to weigh it against a standard supersupercoin consisting of 9 extra coins of standard weight.

Here is another solution for $3 \times c + 1$ coins, and it requires only one extra standard coin. Test $c+1$ of the given coins against c of them plus the one standard coin. If they balance, the culprit (if there is one) is in the remaining c coins, so solve the problem for the remaining c coins. If they don't balance, make c pairs with a coin from the light side and a coin from the heavy side in each pair (one coin is left over). Be sure to mark each coin in each pair to say whether it came from the light side or the heavy side. Now solve the problem for these c pairs. If a pair is identified as being light, then its light coin is the culprit. Similarly if a pair is identified as being heavy, then its heavy coin is the culprit. If all c pairs are equal, then the remaining coin is the culprit, and we already know whether it is light or heavy. This informally described solution works for 1, 4, 7, 10, 13, ... coins, so we'll have to transform it to work on any number of coins.