27 (pirate gold) Islands X and Y contain knights who always tell the truth, knaves who always lie, and possibly some normal people who sometimes tell the truth and sometimes lie. There is gold on at least one of the islands, and the people know which island(s) it is on. You find a message from the pirate who buried the gold, with the following clue (which we take as an axiom): "If there are any normal people on these islands, then there is gold on both islands.". You are allowed to dig on only one island, and you are allowed to ask one question of one random person. What should you ask in order to find out which island to dig on?

After trying the question, scroll down to the solution.

§ Let x mean there is gold on Island X.
Let y mean there is gold on Island Y.
Let k mean the person we ask is a knight.
Let v mean the person we ask is a knave.
Let n mean the person we ask is normal.
Let q be the question we ask (this is what we want to find).
Let a be the answer we receive.
The person we ask is one of knight, knave, or normal:
(0) $(k \lor v \lor n) \land \neg(k \land v) \land \neg(k \land n) \land \neg(v \land n)$
There is gold on at least one of the islands:

$$x \lor y$$

which we will later discover we want in the form
(1) $\neg x \Rightarrow y$
The pirate's axiom tells us
(2) $n \Rightarrow x \land y$
If the person we ask is normal, we have no idea whether the answer is the truth. If the person we ask is not normal, then by axiom (0) the person is a knight or knave, and their answer is the truth ($a{=}q$) if and only if the person a knight:

$$\neg n \ \Rightarrow \ (a{=}q){=}k$$

In a few minutes, I will discover that I need to use the associativity of binary $=$, and rewrite this axiom as
(3) $\neg n \ \Rightarrow \ a{=}(q{=}k)$
Now we want to find a question so that the answer tells us what island to dig on. Let's say if the answer is $\top$ we'll dig on X, and if the answer is $\bot$ we'll dig on Y.

	if a **then** x **else** y **fi**	case idempotent
$=$	**if** n **then if** a **then** x **else** y **fi else if** a **then** x **else** y **fi fi**	context: (2) and (3)
$=$	**if** n **then if** a **then** $\top$ **else** $\top$ **fi else if** $q{=}k$ **then** x **else** y **fi fi**	case idempotent and one case generalization
$=$	$n \ \lor$ **if** $q{=}k$ **then** x **else** y **fi**	(1)
$\Leftarrow$	**if** $q{=}k$ **then** x **else** y **fi**	
$\Leftarrow$	**if** $q{=}k$ **then** x **else** $\neg x$ **fi**	
$=$	$(q{=}k){=}x$	associative law for binary $=$
$=$	$q{=}(k{=}x)$	

So the question could be: "Is it true that you are a knight if and only if there is gold on Island X?". There are other questions that work too.