261 (machine division) Given two natural numbers, write a program to find their quotient using only addition, subtraction, doubling, halving, test for even, and comparisons.

After trying the question, scroll down to the solution.

§ I suppose the question means the natural part of the quotient, discarding the remainder. Let n , m , and q be natural number variables. We want $q' = div\ n\ m$.

$$\begin{aligned} q' = div\ n\ m \ \Leftarrow\ & \textbf{if } n<m \textbf{ then } q:= 0 \\ & \textbf{else } n:= n-m.\ \ q' = div\ n\ m.\ \ q:= q+1 \textbf{ fi} \end{aligned}$$

That solution takes time $div\ n\ m$. Let's try for better.

$$\begin{aligned} q' = div\ n\ m \ \Leftarrow\ & \textbf{if } n<m \textbf{ then } q:= 0 \\ & \textbf{else if } even\ n \textbf{ then } n:= n/2.\ \ q' = div\ n\ m.\ \ q:= q\times 2 \\ & \quad\ \ \textbf{else } n:= n-m.\ \ q' = div\ n\ m.\ \ q:= q+1 \textbf{ fi fi} \end{aligned}$$

That solution takes time something like $log\ n$.