242 (interval union) A collection of intervals along a real number line is given by the list of left ends L and the corresponding list of right ends R. List L is sorted. The intervals might sometimes overlap, and sometimes leave gaps. Write a program to find the total length of the number line that is covered by these intervals.

After trying the question, scroll down to the solution.

§ One approach is to add up the intervals. Another is to add up the gaps. I'll try the first approach, and I'll try a solution of the form

$s{:=}\ 0.$ **for** $i{:=}\ 0;..\#L$ **do** $s{:=}\ s +$ something **od**

using invariant $A\ i$ defined informally as

$s =$ length covered by the intervals represented by $L[0;..i]$ and $R[0;..i]$

and the body of the loop considers one new interval from $L\ i$ to $R\ i$. This new interval may already be totally covered, partly covered, or not covered at all. For $i{>}0$ we can determine which of these three cases we have by knowing which of the previous intervals extended farthest to the right. Suppose it was the interval from $L\ k$ to $R\ k$ for some $k{:}\ 0,..i$.

Totally covered case: $L\ k \le L\ i \le R\ i \le R\ k$. Then $A\ i \Rightarrow A'(i{+}1) \Longleftarrow ok$.

Partly covered case: $L\ k \le L\ i \le R\ k \le R\ i$. We have $L\ k \le L\ i$ because $k{<}i$ and list L is sorted. Then $A\ i \Rightarrow A'(i{+}1) \Longleftarrow s{:=}\ s + R\ i - R\ k$.

Not covered at all case: $L\ k \le R\ k \le L\ i \le R\ i$. Again we have $L\ k \le L\ i$ because $k{<}i$ and list L is sorted. Then $A\ i \Rightarrow A'(i{+}1) \Longleftarrow s{:=}\ s + R\ i - L\ i$.

All three cases can be expressed as $s{:=}\ s + (R\ k){\uparrow}(R\ i) - (R\ k){\uparrow}(L\ i)$.

To keep track of $R\ k$, introduce variable r , and strengthen $A\ i$ with the conjunct

$r =$ farthest right point so far

Now formally, the problem is P where

$P\ =\ s' = \Sigma j{:}\ 0,..\#L\cdot\ {\Uparrow}R[0;..j{+}1] - ({\Uparrow}R[0;..j]){\uparrow}(L\ j)$

and $A\ i$ is defined as

$r = {\Uparrow}R[0;..i]\ \land\ s = \Sigma j{:}\ 0,..i\cdot\ {\Uparrow}R[0;..j{+}1] - ({\Uparrow}R[0;..j]){\uparrow}(L\ j)$

The program is now

$P\ \Longleftarrow\ s{:=}\ 0.\ r{:=}\ {-}\infty.\ A\ 0 \Rightarrow A'(\#L)$

$A\ 0 \Rightarrow A'(\#L)\ \Longleftarrow$ **for** $i{:=}\ 0;..\#L$ **do** $i{:}\ 0,..\#L \land A\ i \Rightarrow A'(i{+}1)$ **od**

$i{:}\ 0,..\#L \land A\ i \Rightarrow A'(i{+}1)\ \Longleftarrow\ s{:=}\ s + r{\uparrow}(R\ i) - r{\uparrow}(L\ i).\ r{:=}\ r{\uparrow}(R\ i)$