

- 24 (caskets) There are two caskets; one is gold and one is silver. In one casket there is a million dollars, and the other is empty. On the gold casket there is an inscription: the money is not in here. On the silver casket there is an inscription: exactly one of these inscriptions is true. Each inscription is either true or false (not both). On the basis of the inscriptions, find the money.

After trying the question, scroll down to the solution.

§ To formalize the problem, we can introduce binary expressions G , g , and s with the following meanings.

G means that the money is in the gold casket and the silver casket is empty.

g means that the inscription on the gold casket is true.

s means that the inscription on the silver casket is true.

We can now express all the given information as axioms about G , g , and s . The inscription on the gold casket is true (g) if and only if ($=$) the money is not in the gold casket ($\neg G$). And the inscription on the silver casket is true (s) if and only if ($=$) exactly one of the inscriptions is true ($s \neq g$). So we have

$$\begin{array}{ll}
 \top & \text{use given information} \\
 = & g = \neg G \wedge s = (s \neq g) \quad = \text{ and } \neq \text{ are mutually associative} \\
 = & g = \neg G \wedge (s = s) \neq g \quad = \text{ is reflexive} \\
 = & g = \neg G \wedge \top \neq g \quad \text{generic axiom to replace } \neq \text{ by } = \neg \\
 = & g = \neg G \wedge \top = \neg g \quad \text{move } \neg, \text{ and } \top \text{ is the identity for } = \\
 = & \neg g = G \wedge \neg g \quad \text{use context } \neg g \\
 = & \top = G \wedge \neg g \quad \top \text{ is the identity for } = \\
 = & G \wedge \neg g
 \end{array}$$

The money is in the gold casket and the silver casket is empty, and the inscription on the gold casket is false. We don't know whether the inscription on the silver casket is true or false.

If you have two caskets with inscriptions on them, and you are told there's money in one of them, the money could be in either casket, regardless of what the inscriptions say. But we are told something more. We are not told whether the inscriptions are true or false, but we are told that the inscriptions are consistent (either true or false, not both true and false). If the money were in the silver casket, then the inscription on the gold casket would be true. Now, if the inscription on the silver casket were true, then it would also be false, and if it were false, then it would also be true. That would be inconsistent. So the money has to be in the gold casket.