

21 (maid and butler) The maid said she saw the butler in the living room. The living room adjoins the kitchen. The shot was fired in the kitchen, and could be heard in all nearby rooms. The butler, who had good hearing, said he did not hear the shot. Given these facts, prove that someone lied. Use the following abbreviations.

mtt = (the maid told the truth)

btt = (the butler told the truth)

blr = (the butler was in the living room)

bnk = (the butler was near the kitchen)

bhs = (the butler heard the shot)

After trying the question, scroll down to the solution.

Solutions

§ The given information, sentence by sentence, is

$$\begin{aligned} & (mtt \Rightarrow blr) \wedge (blr \Rightarrow bnk) \wedge (bnk \Rightarrow bhs) \wedge \underline{(btt \Rightarrow \neg bhs)} && \text{contrapositive} \\ = & (mtt \Rightarrow blr) \wedge (blr \Rightarrow bnk) \wedge (bnk \Rightarrow bhs) \wedge (bhs \Rightarrow \neg btt) && \text{transitivity of } \Rightarrow \\ \Rightarrow & mtt \Rightarrow \neg btt && \text{inclusion (material implication)} \\ = & \neg mtt \vee \neg btt \end{aligned}$$

which says that at least one of them lied.