189 (ternary search) The problem is similar to binary search (Exercise 187). The strategy this time is to identify which third of the list contains the item if it occurs, then which ninth, then which twenty-seventh, and so on.

After trying the question, scroll down to the solution.

§ Let L be the list, let x be the value sought, let p be a binary variable whose final value says whether x is present in L, and let h, i, j, k be indexes. The problem is defined as

$$x: L\,(0,..\#L) \;=\; p' \Longrightarrow L\,h' = x$$

In general, the indexes will be in the order $0 \le h \le i \le j \le k \le \#L$ and the segment still to be searched is $h;..k$ and indexes i and j are at one-third and two-thirds of the way through the segment. So define R as

$$R \;=\; (x: L\,(h,..k) \;=\; p' \Longrightarrow L\,h' = x)$$

Solve the problem as follows.

$$(x: L\,(0,..\#L) \;=\; p' \Longrightarrow L\,h' = x) \;\Longleftarrow\; h{:=}\,0.\;\; k{:=}\,\#L.\;\; h{<}k \Rightarrow R$$

$$h{<}k \Rightarrow R \;\Longleftarrow\; \textbf{if } k{-}h = 1 \textbf{ then } p{:=}\, L\,h = x \;\textbf{else if } k{-}h = 2 \textbf{ then if } L\,h = x \textbf{ then } p{:=}\,\top \textbf{ else } h{:=}\,h{+}1.\;\; p{:=}\,Lh{=}x \textbf{ fi } \textbf{else } k{-}h \ge 3 \Rightarrow R \textbf{ fi fi}$$

$$k{-}h \ge 3 \Rightarrow R \;\Longleftarrow\; i{:=}\,div\,(h{+}k)\,3.\;\; j{:=}\,div\,(h + 2{\times}k)\,3.\;\; \textbf{if } L\,i > x \textbf{ then } k{:=}\,i \textbf{ else if } L\,j \le x \textbf{ then } h{:=}\,j \textbf{ else } h{:=}\,i.\; k{:=}\,j \textbf{ fi fi}.\;\; h{<}k \Rightarrow R$$

The proofs of the refinements are not very different from Subsection 4.2.5. For the timing, let lbt be the logarithm base 3 function.

$$t' \le t + ceil\; lbt\,(\#L) \;\Longleftarrow\; h{:=}\,0.\;\; k{:=}\,\#L.\;\; h{<}k \Rightarrow t' \le t + ceil\; lbt\,(k{-}h)$$

$$h{<}k \Rightarrow t' \le t + ceil\; lbt\,(k{-}h) \;\Longleftarrow\; \textbf{if } k{-}h = 1 \textbf{ then } p{:=}\, L\,h = x \;\textbf{else if } k{-}h = 2 \textbf{ then if } L\,h = x \textbf{ then } p{:=}\,\top \textbf{ else } h{:=}\,h{+}1.\;\; p{:=}\,L\,h = x \textbf{ fi } \textbf{else } k{-}h \ge 3 \Rightarrow t' \le t + ceil\; lbt\,(k{-}h) \textbf{ fi fi}$$

$$k{-}h \ge 3 \Rightarrow t' \le t + ceil\; lbt\,(k{-}h) \;\Longleftarrow\; i{:=}\,div\,(h{+}k)\,3.\;\; j{:=}\,div\,(h + 2{\times}k)\,3.\;\; \textbf{if } L\,i > x \textbf{ then } k{:=}\,i \textbf{ else if } L\,j \le x \textbf{ then } h{:=}\,j \textbf{ else } h{:=}\,i.\;\; k{:=}\,j \textbf{ fi fi}.\;\; h{<}k \Rightarrow t' \le t + ceil\; lbt\,(k{-}h)$$

Again, the proofs are not very different from Subsection 4.2.5.