149 Is the refinement

$$P \Leftarrow \textbf{if } x=0 \textbf{ then } ok \textbf{ else } x:= x-1.\ \ t:= t+1.\ \ P \textbf{ fi}$$

a theorem when

$$P = x<0 \Rightarrow x'=1 \land t'=\infty$$

Is this reasonable? Explain.

After trying the question, scroll down to the solution.

§ Yes, it is a theorem. Proof by cases:

$$
\begin{array}{ll}
 & (x{<}0 \Rightarrow x'{=}1 \land t'{=}\infty) \;\Leftarrow\; x{=}0 \land ok \qquad \text{portation} \\
= & x'{=}1 \land t'{=}\infty \;\Leftarrow\; x{<}0 \land x{=}0 \land ok \\
= & x'{=}1 \land t'{=}\infty \;\Leftarrow\; \bot \\
= & \top
\end{array}
$$

$$
\begin{array}{ll}
 & (x{<}0 \Rightarrow x'{=}1 \land t'{=}\infty) \;\Leftarrow\; x{\neq}0 \land (x{:=}\,x{-}1.\;\; t{:=}\,t{+}1.\;\; x{<}0 \Rightarrow x'{=}1 \land t'{=}\infty) \\
 & \qquad \text{portation and two substitutions} \\
= & x'{=}1 \land t'{=}\infty \;\Leftarrow\; x{<}0 \land (x{-}1{<}0 \Rightarrow x'{=}1 \land t'{=}\infty) \qquad \text{discharge} \\
= & x'{=}1 \land t'{=}\infty \;\Leftarrow\; x{<}0 \land x'{=}1 \land t'{=}\infty \qquad \text{specialization} \\
= & \top
\end{array}
$$

When $x{<}0$ the execution time is infinite ($t'{=}\infty$) so there is no final state. It is therefore somewhat unreasonable to say $x'{=}1$. On the other hand, no observation can ever show otherwise.