

145 Let  $x$  be an integer variable and let  $P \equiv x=0$ . Prove the refinement

$$P \Leftarrow \begin{array}{l} \mathbf{if } x > 0 \mathbf{ then } x := x - 1. P \\ \mathbf{else if } x < 0 \mathbf{ then } x := x + 1. P \\ \mathbf{else ok fi fi} \end{array}$$

After trying the question, scroll down to the solution.

§ Using refinement by cases, I must prove three things:

$$x'=0 \iff x>0 \wedge (x:=x-1. x'=0)$$

$$x'=0 \iff x<0 \wedge (x:=x+1. x'=0)$$

$$x'=0 \iff x=0 \wedge ok$$

Let's start with the first.

$$x>0 \wedge (x:=x-1. x'=0)$$

$$= x>0 \wedge x'=0$$

$$\implies x'=0$$

use substitution law  
specialization

Now the middle one.

$$x<0 \wedge (x:=x+1. x'=0)$$

$$= x<0 \wedge x'=0$$

$$\implies x'=0$$

use substitution law  
specialization

And the last one.

$$x=0 \wedge ok$$

$$= x=0 \wedge x'=x$$

$$\implies x'=0$$

replace *ok*  
transitivity