143 In natural variables s and n prove

$$P \Leftarrow \textbf{if } n=0 \textbf{ then } ok \textbf{ else } n:=n-1.\ s:=s+2^n-n.\ t:=t+1.\ P \textbf{ fi}$$

where $P = s' = s + 2^n - n\times(n-1)/2 - 1 \ \wedge\ n'=0 \ \wedge\ t' = t+n$.

After trying the question, scroll down to the solution.

§ Proof by parts (3 of them) and by cases (2 of them), so 6 things to prove. First part, first case, starting with the right side:

$n{=}0 \land ok$ — expand ok

$= \quad n{=}0 \land n'{=}n \land s'{=}s \land t'{=}t$ — arithmetic and specialization

$\Rightarrow \quad s' = s + 2^n - n{\times}(n-1)/2 - 1$

First part, last case:

$n{>}0 \land (n{:=}\ n-1.\ \ s{:=}\ s{+}2^n{-}n.\ \ t{:=}\ t{+}1.\ \ s' = s + 2^n - n{\times}(n-1)/2 - 1)$ — Substitution Law 3 times

$= \quad n{>}0 \ \land\ s' = s + 2^{n-1} - (n-1) + 2^{n-1} - (n-1){\times}(n-1-1)/2 - 1$ — arithmetic

$= \quad n{>}0 \ \land\ s' = s + 2^n - n{\times}(n-1)/2 - 1$ — specialization

$\Rightarrow \quad s' = s + 2^n - n{\times}(n-1)/2 - 1$

Middle part, first case:

$n{=}0 \land ok$ — expand ok

$= \quad n{=}0 \land n'{=}n \land s'{=}s \land t'{=}t$ — transitivity and specialization

$\Rightarrow \quad n'{=}0$

Middle part, last case:

$n{>}0 \land (n{:=}\ n-1.\ \ s{:=}\ s{+}2^n{-}n.\ \ t{:=}\ t{+}1.\ \ n'{=}0)$ — Substitution Law 3 times

$= \quad n{>}0 \ \land\ n'{=}0$ — specialization

$\Rightarrow \quad n'{=}0$

Last part, first case:

$n{=}0 \land ok$ — expand ok

$= \quad n{=}0 \land n'{=}n \land s'{=}s \land t'{=}t$ — arithmetic and specialization

$\Rightarrow \quad t' = t{+}n$

Last part, last case:

$n{>}0 \land (n{:=}\ n-1.\ \ s{:=}\ s{+}2^n{-}n.\ \ t{:=}\ t{+}1.\ \ t' = t{+}n)$ — Substitution Law 3 times

$= \quad n{>}0 \ \land\ t' = t{+}1{+}n{-}1$ — arithmetic and specialization

$\Rightarrow \quad t' = t{+}n$