

141 Let s and n be number variables. Let Q be a specification defined as

$$Q = s' = s + n \times (n-1) / 2$$

(a) Prove the refinement

$$Q \Leftarrow n := n-1. s := s+n. Q$$

(b) Add time according to the recursive measure, replace Q by an implementable timing specification, and reprove the refinement.

After trying the question, scroll down to the solution.

$$\begin{aligned}
\text{\S(a)} & \quad n:=n-1. s:=s+n. Q && \text{expand } Q \\
= & \quad n:=n-1. s:=s+n. s' = s+n \times (n-1)/2 && \text{substitution law twice} \\
= & \quad s' = s+n-1+(n-1) \times (n-1-1)/2 && \text{arithmetic} \\
= & \quad s' = s+n \times (n-1)/2 \\
= & \quad Q
\end{aligned}$$

$$\begin{aligned}
\text{\S(b)} & \quad t'=\infty \iff n:=n-1. s:=s+n. t:=t+1. t'=\infty \\
= & \quad n:=n-1. s:=s+n. t:=t+1. t'=\infty && \text{substitution law three times} \\
= & \quad t'=\infty
\end{aligned}$$

OR

$$\begin{aligned}
& \quad t' \geq t \iff n:=n-1. s:=s+n. t:=t+1. t' \geq t \\
= & \quad n:=n-1. s:=s+n. t:=t+1. t' \geq t && \text{substitution law three times} \\
= & \quad t' \geq t+1 \\
\Rightarrow & \quad t' \geq t
\end{aligned}$$