14 Complete the following laws of Binary Theory

(a) $\top =$

(b) $\bot =$

(c) $\neg a =$

(d) $a \wedge b =$

(e) $a \vee b =$

(f) $a = b =$

(g) $a \neq b =$

(h) $a \Rightarrow b =$

by adding a right side using only the following symbols (in any quantity)

(i) $\neg \wedge a\ b\ (\ )$

(ii) $\neg \vee a\ b\ (\ )$

(iii) $\neg \Rightarrow a\ b\ (\ )$

(iv) $\neq \Rightarrow a\ b\ (\ )$

(v) $\neg$ **if then else fi** $a\ b\ (\ )$

That's $8 \times 5 = 40$ questions.

After trying the question, scroll down to the solution.

§	(i)	(ii)	(iii)	(iv)	(v)
(a)	$\neg(a \wedge \neg a)$	$a \vee \neg a$	$a \Rightarrow a$	$a \Rightarrow a$	**if** a **then** a **else** $\neg a$ **fi**
(b)	$a \wedge \neg a$	$\neg(a \vee \neg a)$	$\neg(a \Rightarrow a)$	$a \neq a$	**if** a **then** $\neg a$ **else** a **fi**
(c)	$\neg a$	$\neg a$	$\neg a$	$a \Rightarrow (a \neq a)$	$\neg a$
(d)	$a \wedge b$	$\neg(\neg a \vee \neg b)$	$\neg(a \Rightarrow \neg b)$	$(a \Rightarrow (a \neq b)) \Rightarrow (a \neq a)$	**if** a **then** b **else** a **fi**
(e)	$\neg(\neg a \wedge \neg b)$	$a \vee b$	$\neg a \Rightarrow b$	$(a \Rightarrow (a \neq a)) \Rightarrow b$	**if** a **then** a **else** b **fi**
(f)	$\neg(a \wedge \neg b)$ $\wedge \neg(\neg a \wedge b)$	$\neg(a \vee b)$ $\vee \neg(\neg a \vee \neg b)$	$\neg((a \Rightarrow b)$ $\Rightarrow \neg(b \Rightarrow a))$	$(a \neq b) \Rightarrow (a \neq a)$	**if** a **then** b **else** $\neg b$ **fi**
(g)	$\neg(a \wedge b)$ $\wedge \neg(\neg a \wedge \neg b)$	$\neg(a \vee \neg b)$ $\vee \neg(\neg a \vee b)$	$(a \Rightarrow b) \Rightarrow \neg(b \Rightarrow a)$	$a \neq b$	**if** a **then** $\neg b$ **else** b **fi**
(h)	$\neg(a \wedge \neg b)$	$\neg a \vee b$	$a \Rightarrow b$	$a \Rightarrow b$	**if** a **then** b **else** $\neg a$ **fi**

Note: using continuing operators, we can write (f)(iii) and (f)(iv) as $a \Rightarrow b \Rightarrow a$.