129 Prove

(a) P and Q are each refined by R if and only if their conjunction is refined by R.

(b) $P \Rightarrow Q$ is refined by R if and only if Q is refined by $P \wedge R$.

After trying the question, scroll down to the solution.

(a) P and Q are each refined by R if and only if their conjunction is refined by R.

§ $(\forall \sigma, \sigma' \cdot P \Leftarrow R) \ \wedge \ (\forall \sigma, \sigma' \cdot Q \Leftarrow R)$ Splitting Law

$= \quad \forall \sigma, \sigma' \cdot (P \Leftarrow R) \ \wedge \ (Q \Leftarrow R)$ Distributive Law

$= \quad \forall \sigma, \sigma' \cdot P \wedge Q \Leftarrow R$

(b) $P \Rightarrow Q$ is refined by R if and only if Q is refined by $P \wedge R$.

§ $\forall \sigma, \sigma' \cdot (P \Rightarrow Q) \Leftarrow R$ Portation

$= \quad \forall \sigma, \sigma' \cdot Q \Leftarrow P \wedge R$