

128 Prove

$$(R \Leftarrow P. \text{if } b \text{ then } ok \text{ else } R \text{ fi}) \wedge (W \Leftarrow \text{if } b \text{ then } ok \text{ else } P. W \text{ fi}) \\ \Leftarrow (R \Leftarrow P. W) \wedge (W \Leftarrow \text{if } b \text{ then } ok \text{ else } R \text{ fi})$$

After trying the question, scroll down to the solution.

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Starting with the bottom side,

$(R \Leftarrow P. W) \wedge (W \Leftarrow \mathbf{if\ } b \mathbf{\ then\ } ok \mathbf{\ else\ } R \mathbf{\ fi})$ idempotent law

$= (R \Leftarrow P. W) \wedge (R \Leftarrow P. W) \wedge (W \Leftarrow \mathbf{if\ } b \mathbf{\ then\ } ok \mathbf{\ else\ } R \mathbf{\ fi})$

In the leftmost conjunct, strengthen W by using the rightmost conjunct.

This strengthens the antecedent of an implication, and so weakens the implication.

$\Rightarrow (R \Leftarrow P. \mathbf{if\ } b \mathbf{\ then\ } ok \mathbf{\ else\ } R \mathbf{\ fi}) \wedge (R \Leftarrow P. W) \wedge (W \Leftarrow \mathbf{if\ } b \mathbf{\ then\ } ok \mathbf{\ else\ } R \mathbf{\ fi})$

In the rightmost conjunct strengthen R by using the middle conjunct.

This strengthens the antecedent of an implication, and so weakens the implication.

$\Rightarrow (R \Leftarrow P. \mathbf{if\ } b \mathbf{\ then\ } ok \mathbf{\ else\ } R \mathbf{\ fi}) \wedge (R \Leftarrow P. W) \wedge (W \Leftarrow \mathbf{if\ } b \mathbf{\ then\ } ok \mathbf{\ else\ } P. W \mathbf{\ fi})$

Drop the middle conjunct.

$\Rightarrow (R \Leftarrow P. \mathbf{if\ } b \mathbf{\ then\ } ok \mathbf{\ else\ } R \mathbf{\ fi}) \wedge (W \Leftarrow \mathbf{if\ } b \mathbf{\ then\ } ok \mathbf{\ else\ } P. W \mathbf{\ fi})$