

114 We can express “there is a smallest natural number” as follows:

$$\exists n: \text{nat} \cdot \forall m: \text{nat} \cdot n \leq m$$

- (a) Now how do we say “Denote that smallest natural number by 0 .” formally? In other words, how do we say “Let's call that smallest natural number 0 .” formally?
- (b) Prove that there are not two different natural numbers that are tied for smallest.

After trying the question, scroll down to the solution.

§(a) $0: \text{nat} \wedge \forall m: \text{nat} \cdot 0 \leq m$

§(b) Let a and b be smallest natural numbers.

$a: \text{nat} \wedge (\forall m: \text{nat} \cdot a \leq m) \wedge b: \text{nat} \wedge (\forall m: \text{nat} \cdot b \leq m)$

Specialize the first \forall with b for m and specialize the last \forall with a for m .

$\Rightarrow a \leq b \wedge b \leq a$

Now, from a generic law (antisymmetry) we have

$= a = b$