113 Let all variables range over the reals. Prove

$$n=m \;=\; \forall k\cdot\ (k\le n)=(k\le m)$$

After trying the question, scroll down to the solution.

§ The easiest proof uses the Connection Law twice. Starting with the left side,

	$n=m$	generic antisymmetry
$=$	$n\le m \land m\le n$	Connection Law twice
$=$	$(\forall k\cdot k\le n \Rightarrow k\le m) \land (\forall k\cdot k\le m \Rightarrow k\le n)$	Splitting Law
$=$	$\forall k\cdot (k\le n \Rightarrow k\le m) \land (k\le m \Rightarrow k\le n)$	antisymmetry (double implication)
$=$	$\forall k\cdot (k\le n)=(k\le m)$	

Another proof, without the Connection Law, goes like this. Starting with the right side,

	$\forall k\cdot (k\le n)=(k\le m)$	idempotence
$=$	$(\forall k\cdot (k\le n)=(k\le m)) \land (\forall k\cdot (k\le n)=(k\le m))$	specialize the left conjunct with n and specialize the right conjunct with m
$\Rightarrow$	$(n\le n)=(n\le m) \land (m\le n)=(m\le m)$	generic reflexive law twice
$=$	$\top=(n\le m) \land (m\le n)=\top$	identity law twice
$=$	$n\le m \land m\le n$	generic antisymmetry
$=$	$n=m$	

That proves $n=m \Leftarrow \forall k\cdot (k\le n)=(k\le m)$. Now we need the other direction.

	$n=m \Rightarrow \forall k\cdot (k\le n)=(k\le m)$	context
$=$	$n=m \Rightarrow \forall k\cdot (k\le n)=(k\le n)$	reflexive
$=$	$n=m \Rightarrow \forall k\cdot \top$	identity
$=$	$n=m \Rightarrow \top$	base
$=$	$\top$	

Now antisymmetry (double implication) gives us the desired result.