CSC465/2104 Test 2 2024 November 14 4:10pm

1 page, 3 questions, 42 marks, 50 minutes
Aids allowed: one letter-sized page, both sides
and the laws from the textbook, 14 pages

The value of each question is indicated in square brackets.
A blank answer is worth about one-third of the marks;
to that, marks will be added for readable and relevant and correct information,
and marks will be subtracted for unreadable or irrelevant or incorrect information.

1 Let s and n be *nat* variables. Here is a refinement.

$$s' = s + 2^n - 1 \quad \Longleftarrow \quad \textbf{if } n{=}0 \textbf{ then } ok \textbf{ else } n{:=}\, n{-}1.\ \ s{:=}\, s + 2^n.\ \ s' = s + 2^n - 1 \textbf{ fi}$$

(a)[12] Prove it.

(b)[3] Insert appropriate time increments according to the recursive measure, and write appropriate timing specifications.

(c)[6] Prove the timing refinement from part (b).

2[9] Let S be a bunch of strings. Using construction and induction, define T to be the bunch of all strings formed by joining together any number of any strings in S in any order. Do not use the * operator; in effect, you are defining the * operator.

3[12] Let i be an extended integer variable, and let t be an extended natural time variable. Let P be a specification such that

$$P \quad \Longleftarrow \quad \textbf{if } i{=}0 \textbf{ then } ok \textbf{ else } i{:=}\, i{-}1.\ \ t{:=}\, t{+}1.\ \ P \textbf{ fi}$$

What solution for P does recursive construction give when we start with $P_0 = t{:=}\, \infty$? Find it, but you do not need to prove that it is a solution.