Getting started on Assignment 1

May 17, 2004

Here are some ideas that may help you get started on Assignment 1. Work these through with your assignment partner (if you're working in a pair), and with your TA.

For all questions, you should sketch the steps you think a valid proof should have. This requires you to think of the proof method you will use (direct proof, contrapositive, contradiction, proof by parts, induction), plus what smaller facts you will need to prove.

You should try to re-express the problem you are trying to solve in your own words. If you make a conjecture, you should try to express it as precisely as possible, and check whether there are any special cases where the conjecture obviously doesn't hold.

Don't assume every problem is solved using induction, since several proof techniques are mentioned in this course. Try more than one, if necessary.

1. Draw pictures of how many segments are in a small one-dimensional lattice. Make a conjecture, and try proving it. In part (b), draw pictures for small (2×2, 3×3) lattices.

2. Drawing pictures is harder in three dimensions. Try to see whether there is a connection with the previous question you can use. Make a conjecture about small cases ($2 \times 2 \times 2$, $3 \times 3 \times 3$). Try to see how the general problem of counting the number of rectangles can be broken into smaller problems.

3. Try small cases (1–4 point or lines), and make a conjecture. See whether there is a connection between parts (a) and (b). Make sure you are clear on the problem being solved, by re-stating it in your own words.

4. Unless you are a gifted artist, it is difficult to draw examples. Try to see how the solutions to the previous part are "contained" in the planes to this part.

5. Using Well-Ordering Principle suggests that we're looking for some subset of the natural numbers. Suppose there is a solution for integers a and b — does this generate some subsets of the natural numbers that you can consider?

6. Try some binary and ternary numbers with only a few digits to see whether the claim makes sense. Is there some way to reduce the claim to a smaller case (useful if you prove this by induction)?

7. Very that the claims work in base 9 and base 11 by checking cases with just a few digits. Is there some way to break the problem up into parts so that you can use the induction hypothesis on just one part?