CSC236, week 2 tutorial hints

May 25, 2004

Here are some further hints meant to help with assignment 1.

Questions 1–3: Read Example 1.1 (pages 22 and 23 of the Course Notes), which proves that the sum of the first n integers is $n(n+1)/2$. Try to to find a closed expression for the sum of the first n squares and the first n cubes.

Question 5: Read Theorem 5.13 (page 128 of the Course Notes). Modify this proof so that it uses well-ordering, in the following way. Without assuming that m and n are in least terms, show that if m and n are natural numbers such that $m/n = \sqrt{2}$, then $m/2$ and $n/2$ are also natural numbers, as are $m/4$ and $n/4$. Iterate this reasoning to produce a non-empty set of natural numbers with no least element (a contradiction).

Questions 6–7: Turn this sketch into a proof that for each odd natural number n, $4^n = 4 \pmod 5$

Base case: Verify that the claim works for the base case, when $n = 1$.

Induction step: If the claim holds for the n, an odd number, show that the claim holds for $n + 2$.

Part of the challenge is writing up the proof is dealing with notation for odd numbers, so that each induction step increments by 2 rather than 1.