A1 help, part 4

In addition to things I have said in hints you may assume without proof, there may be facts from other branches of mathematics (e.g. geometry) that you find are too far from the content of this course to prove from scratch, so in your solution you may want to say you are assuming these without proof. In general, the marker will give you some credit for identifying your assumptions, and will then make a judgment about whether you are assuming something you should have been able to prove using things you've learned in this course.

QUESTION 4: You can probably visualize how many parts one plane divides space into, as well as how many parts two and three planes divide space into (do you get eight parts for three planes?).

For more than three planes, it probably gets hard to visualize, so you need to count more abstractly. Suppose you've already divided up three-dimensional space using $n-1$ planes. When you add the nth plane, its intersections with the previous $n-1$ planes are lines, and its intersections with the 3-D regions bounded by the previous planes are 2-D regions bounded by those lines.

If you assume that the nth plane divides each of the previous regions it encounters into only two parts (to prove this may require some reasoning about convexity, which is beyond the scope of this course), then the number of new 3-D regions created corresponds to the number of 2-D regions the nth plane is divided into. So, to maximize the number of new 3-D regions is equivalent to maximizing the 2-D regions the nth plane is divided into. So the solution to 3(b) becomes part of the solution to 4.

QUESTION 5: If you want to restrict your attention to natural numbers a and b, you can show that any integer solution to $a/b = b/(a-b)$ implies there is a solution in natural numbers, simply by putting absolute values around the ratios and thinking about the implications.

Suppose you have decided that you want to prove there are no natural numbers a and b that satisfy the given equation. One way to proceed would be to show that the set of natural numbers that take the role of either the numerator or denominator in such an equation is empty. If the set were non-empty, Well-Ordering says it has a smallest member. Think about how you could contradict this.