CSC236, Summer 2004, Assignment 5

Due: Thursday, August 12, 10 pm

Instructions

Please work on all questions. Turn in the outline and structure of a proof, even if you cannot provide every step of the proof, and we will try to assign part marks. However, if there is any question you cannot see how to even begin, leave it blank you you will receive 20% of the marks for that question.

Be sure to give full credit to any sources you consult (other than course notes, TAs, and the instructor) in preparing this problem set. If you try to pass off somebody else's work as your own for credit, you are committing an academic offense, and that can entail serious consequences. Any ideas that you do not attribute to someone else are assumed to be the ideas of the author(s) listed below, and will be evaluated for grading.

Write your name(s) and student number(s) (maximum of two names and two student numbers) in the space below, and staple this page to the front of your assignment.

Name ______________________________

Student # ______________________________

Name ______________________________

Student # ______________________________

1. Prove or disprove the following claims, assuming R, S, and T are regular expressions.

   (a) If $L(R^*) = \mathrm{Rev}(L(R^*))$ then $L(R) = \mathrm{Rev}(L(R))$.

   (b) If $L(R) = \mathrm{Rev}(L(R))$ then $L(R^*) = \mathrm{Rev}(L(R^*))$.

   (c) If $(RS)^* \equiv (R^*S^*)$ then $R \equiv S$.

   (d) If $R \equiv RR$ and $R \not\equiv \emptyset$, then $R \equiv R^*$.

2. Give a regular expressions that denotes L, and justify your answer.

   (a) $L = \{x \in \{0,1\}^* : x \text{ contains at least four 0s}\}$.

   (b) $L = \{x \in \{0,1\}^* : x \text{ contains at least two 0s and at most one 1}\}$

   (c) $L = \{x \in \{0,1\}^* : x \text{ contains an odd number of 0s, or exactly two 1s}\}$

   (d) $L = \{x \in \{0,1\}^* : x \text{ doesn't contain the substring 101}\}$

   (e) $L = \{x \in \{0,1\}^* : x \text{ is neither 11 nor 111}\}$

3. For each of the following languages, L, construct a DFSA that accepts L and a regular expression that denotes L. Prove your automata and regular expressions are correct.

   (a) $L = \{x \in \{0,1\}^* : |x| > 2 \text{ or } x \text{ contains suffix 1}\}$

   (b) $L = \{x \in \{0,1\}^* : x \text{ contains substring 11 and } x \text{ has an even number of 0s}\}$

   (c) Let $(x)_2$ denote the value of x as a binary number.

   $$L = \{x \in \{0,1\}^* : \text{ for some } n \in \mathbb{N}, (x)_2 = n \text{ and } \text{ for some } i, j \in \mathbb{N}, (n \operatorname{div} 2^i) \bmod 2^j = 5\}$$

4. Let

$$\begin{aligned} L_1 &= \{x \in \{0,1\}^* : \text{ for some } k \in \mathbb{N}, x \text{ has } 3k+2 \text{ zeros}\} \\ L_2 &= L(11(0+1)^*). \end{aligned}$$

   (a) Construct DFSAs M_1 and M_2 so that $L_1 = L(M_1)$ and $L_2 = L(M_2)$.

   (b) Use the Cartesian product construction (page 228) to create a DFSA M' that accepts $L_1 \cap L_2$.

   (c) Give a state invariant for M', and prove it correct.

   (d) Use the previous part to prove that $L(M') = L_1 \cap L_2$.

5. Is L regular? Justify your claim.

   (a) L is the language of first-order formulas with variables $\{x_1, x_2, \ldots\}$, predicate symbol S of arity 3, and constant symbol $\mathbf{c}$.

   (b) $L = \{x \in \{0,1\}^* : |x| \text{ is prime}\}$

   (c) $L = \{x \in \{0,1\}^* : x \text{ contains exactly one 1 and } x \text{ contains an even number of 0s }\}$.

   (d) $L = \{x \in L(0^n 1 0^n) : n \in \mathbb{N}, \Sigma = \{0,1\}\}$

   (e) $L = \{x \in \{0,1\}^* : x \text{ contains an equal number of strings 01 and 10}\}$