Assignment 2 hints, second portion

QUESTION 1: If you call the number of unlabelled binary trees with n nodes $B(n)$ (or whatever you want to name this function), then you can break up $B(n)$ into

- the number with n nodes and a left sub-tree with zero nodes, plus
- the number with n nodes and a left sub-tree with one node, plus
- $\vdots$
- the number with n nodes and a left sub-tree with k nodes ($0 \leq k \leq n-1$), plus
- $\vdots$
- the number with n nodes and a left sub-tree with $n-1$ nodes.

You can make this expression precise using the $\sum$ notation.

Now you need to prove that each term in your expression is equal to some expression involving $B(k)$, where $k < n$. When you succeed, you will have proved that $B(n)$ is the sum of expressions that involve $B(k)$, where $k < n$. This is the step where you need to concentrate your proof-making skill.

You may find this approach unfamiliar (but unfamiliarity is occasionally a good thing), since in your proof you will focus on a typical term of your sum, rather than on manipulating the whole sum.