#### Multilevel/Hierarchical Models II



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# Item-response (Rasch) model

- J persons, K items
- $y_i = 1$  if the response is correct
- Model:  $P(y_i = 1) = \sigma(\alpha_{j[i]} \beta_{k[i]})$ 
  - $a_j$  is the ability of person j
  - $\beta_k$  is the difficulty of problem k
- Non-identifyable: can increase the alphas and the betas by a constant and get the same probabilities
  - Can subtract the mean alpha to deal with this

### Multilevel model

- $\alpha_j \sim N(0, \sigma_{\alpha}^2)$
- $\beta_j \sim N(\mu_\beta, \sigma_\beta^2)$
- $\mu_{\alpha}$  set to 0 to avoid non-identifyability

# Item specific "discrimination" parameter

•  $P(y_i = 1) = \sigma(\gamma_{k[i]}(\alpha_{j[i]} - \beta_{k[i]}))$ 



Figure 14.14 Curves and simulated data from the logistic item-response (Rasch) model for items k with "difficulty" parameter  $\beta_k = 1$  and high, low, zero, and negative "discrimination" parameters  $\gamma_k$ .

- $P(y_i = 1) = \sigma(\gamma_{k[i]}(\alpha_{j[i]} \beta_{k[i]}))$
- Identifyability problems?

#### Stroop task

Stroop Task 1

READ THE WORDS

RED BLUE BLUE RED GREEN GREEN RED GREEN GREEN BLUE BLUE GREEN Stroop Task 2 SAY THE COLOUR OF THE INK XXXX Stroop Task 3 SAY THE COLOUR OF THE INK

RED	BLUE
BLUE	RED
GREEN	GREEN
RED	GREEN
GREEN	BLUE
BLUE	GREEN

#### Strawman model

- $y_{ij} = \beta_0 + \beta_1 X_{ij} + e_{ij}, e_{ij} \sim N(0, \sigma_e^2)$ 
  - $y_{ij}$ : reaction time of of i-th subject, j-trial
  - *X<sub>ij</sub>*: congruent/incongruent condition for i-th subject j-th trial

#### Model 2

• 
$$y_{ij} = \beta_0 + \beta_1 X_{ij} + u_{0i} + u_{1i} X_{ij} + e_{ij}$$
  
 $u_{0i} \sim N(0, \sigma_{u0}^2)$   
 $u_{1i} \sim N(0, \sigma_{u1}^2)$   
 $e_{ij} \sim N(0, \sigma_e^2)$ 



# Yarkoni's argument #1

- In the first model, rejecting  $\beta_1 = 0$  means that it is unlikely for the particular subjects we observe that there was no difference between congruent/noncongruent conditions
- In the second model, rejecting  $\beta_1 = 0$  means that for subjects as modeled by Model 2, it is unlikely that there was no difference between congruent/non-congruent conditions

# Yarkoni's argument #2

- Research subjects are not the only random effects: so are stimuli, experimenters, research sites, etc.
- Stimuli as non-random effects
  - Strictly speaking, any specific experiment shows that *the particular stimuli used* have an effect