Week 4: notes

June 5, 2014

1 Sample detailed proofs

1.1 $[3|n] \Rightarrow [3|n^2]$

Problem: Prove or disprove the claim

$$[3|n] \Rightarrow [3|n^2]$$

$m|n$ (sometimes pronounced "m divides n) means that n is divisible by m. In other words,

$$[m|n] \Leftrightarrow [\exists k \in \mathbb{Z}, n = km]$$

Solution:
The claim is **true**. Proof:

1. Let k be an integer such that $n = 3k$ (k exists since $3|n$)
2. Then $n^2 = 9k^2 = 3(3k^2)$ (refactoring)
3. $3(3k^2)$ is divisible by 3 ($3k^2$ is an integer)
4. Therefore, n^2 is divisible 3

1.2 $[8|n^2] \Rightarrow [8|n]$

Problem: Prove or disprove the claim

$$[8|n^2] \Rightarrow [8|n]$$

The claim is **false**. Proof:

Let $n = 4$. $8|4^2$ (since $4^2 = 2*8$) but $\neg(8|4)$ (since 4 is a positive integer that's smaller than 8). So for $n = 4$, $[8|n^2] = T$, and $[8|n] = F$, so $[[8|n^2] \Rightarrow [8|n]] = F$.

1.3 $[3|n^2] \Rightarrow [3|n]$

Problem: Prove or disprove the claim. For all $n \in \mathbb{N}$

$$[3|n^2] \Rightarrow [3|n]$$

The claim is **true**. Proof:

1. Let k be an integer such that $n^2 = 3k$ (k exists since $3|n^2$)

2. $n^2 = 3^{k_3} * p_1^{k_{p_1}} * p_2^{k_{p_2}} * .... * p_m^{k_{p_m}}$, where p_1, p_2, ... p_m are different prime factors of n^2 (every number can factorized into prime factors in a unique way)

3. n has the same prime factors as n^2 (since $n^2 = n * n$) so $n = 3^{k_3/2} * q$ where q is a product of prime factors of n^2

4. Therefore, n is divisible by 3.

5. So if we assume that $3|n^2$, we conclude that $3|n$.

6. So $[3|n^2] \Rightarrow [3|n]$