

As in Tutorial 1, suppose that you are given seven different programs  $T, U, V, W, X, Y, Z$ , each meant to solve the same task, where programs  $T, V, X$  are written in Java and programs  $U, W, Y, Z$  are written in Python (not Java). Let  $P$  represent the set of all programs (our “universe” or “domain”),  $J$  represent the set of all Java programs, and  $C$  represent the set of all correct programs. (Assume that these are the only seven programs that exist for solving the task.)

Recall that in class, we have seen how set notation like “ $x \in C$ ” can be expressed in predicate notation as “ $C(x)$ ”, and how this can be used to write different sentences symbolically. Make sure that you understand this correspondence well before answering the following questions.

- For each English sentence below, give representation(s) of the sentence that use the language of symbolic logic. In this course, we prefer that you use quantifiers over the whole universe (in this case  $P$ ) and then use predicate notation to restrict the domain.

NOTE: We show some variants for some of the answers. Keep in mind that these do not constitute all possible correct answers.

- Some incorrect program is written in Java.

Preferred:  $\exists x \in P, \neg C(x) \wedge J(x)$

Alternate:  $\exists x \in \bar{C}, J(x)$  or  $\exists x \in J, \neg C(x)$

- No Java program is correct.

Preferred:  $\neg(\exists x \in P, J(x) \wedge C(x))$  or  $\forall x \in P, J(x) \Rightarrow \neg C(x)$

Alternate:  $\forall x \in J, \neg C(x)$  or  $\forall x \in C, \neg J(x)$

- Only programs written in Python are incorrect.

Preferred:  $\forall x \in P, \neg C(x) \Rightarrow \neg J(x)$

or the contrapositive  $\forall x \in P, J(x) \Rightarrow C(x)$

Alternate:  $\forall x \in \bar{C}, \neg J(x)$  or  $\forall x \in J, C(x)$

- The program is correct and is written in Python.

Preferred:  $C(x) \wedge \neg J(x)$

Alternate:  $x \in C \cap \bar{J}$

- If some Java program is correct, then all Java programs are correct.

Preferred:  $(\exists x \in P, J(x) \wedge C(x)) \Rightarrow (\forall x \in P, J(x) \Rightarrow C(x))$

Alternate:  $(\exists x \in J, C(x)) \Rightarrow (\forall x \in J, C(x))$

Note that parentheses are necessary to limit the scope of  $x$ .

2. Give a *natural* English sentence that captures the meaning of each symbolic sentence below.

NOTE: We show some variants for some of the answers. Keep in mind that these do not constitute all possible correct answers.

(a)  $\exists x \in P, \neg J(x) \wedge C(x)$

*Some Python program is correct.*

*Some correct program is written in Python.*

*There is some program written in Python that is correct.*

(b)  $\forall x \in P, \neg J(x) \wedge C(x)$

*Every program is written in Python and is correct.*

*All programs are correct and written in Python.*

(c)  $\neg \forall x \in P, C(x) \Rightarrow J(x)$

*Not every correct program is written in Java.*

*Some correct program is written in Python.*

(d)  $\forall x \in P, \neg J(x) \Leftrightarrow C(x)$

*Programs are correct if and only if they are written in Python.*

*A program is correct exactly when it is written in Python.*

*All Python programs are correct and all correct programs are written in Python.*

(e)  $(\forall x \in P, J(x) \Rightarrow C(x)) \vee (\forall x \in P, J(x) \Rightarrow \neg C(x))$

*Either all Java programs are correct, or all Java programs are incorrect.*

*(Note how the statement above is different from “All Java programs are either correct or incorrect.” This states that each Java program can be correct or incorrect independently of the other Java programs—it would be true if some Java programs were correct and others were incorrect. However, the statement above expresses that the same property must hold for all Java programs—it would be false if some Java programs were correct and others were incorrect.)*