

#### **Statistical Machine Translation**

- Challenges to statistical machine translation
- Sentence alignment
- IBM model
- Phrase-based translation
- Decoding
- Evaluation



#### How to use the noisy channel

• How does this work?

$$E^* = \underset{E}{\operatorname{argmax}} P(F|E)P(E)$$

- P(E) is a **language model** (e.g., N-gram) and encodes knowledge of word order.
- P(F|E) is a word-level translation model that encodes only knowledge on an *unordered* word-by-word basis.
- Combining these models can give us naturalness and fidelity, respectively.

#### Sentence alignment

- Sentences can also be unaligned across translations.
  - E.g., He was happy.<sub>E1</sub> He had bacon.<sub>E2</sub>  $\rightarrow$  Il était heureux parce qu'il avait du bacon.<sub>F1</sub>

$E_1$	$F_1$		$E_1$	$F_1$
$E_2$	$F_2$		$E_2$	
$E_3$	$F_3$		$E_3$	$F_2$
$E_4$	$F_4$		$E_4$	$F_3$
$E_5$	$F_5$		$E_5$	$F_4$
$E_6$	$F_6$	,		$F_5$
$E_7$	$F_7$		$E_6$	$F_6$
•••			$E_7$	$\overline{F_7}$



#### Sentence alignment

- We often need to align sentences before we can align words.
- We'll look at two broad classes of methods:
  - 1. Methods that only look at sentence length,
  - 2. Methods based on lexical matches, or "cognates".



## 1. Sentence alignment by length

$E_1$	$F_1$
$E_2$	
$E_3$	$F_2$
$E_4$	$F_3$
$E_5$	$F_4$
	$F_5$
$E_6$	$F_6$

It's a bit more complicated – see paper on course webpage

We can associate costs with different **types** of alignments.

 $C_{i,j}$  is the prior cost of aligning i sentences to j sentences.

$$Cost = Cost(\mathcal{L}_{E_{1}} + \mathcal{L}_{E_{2}}, \mathcal{L}_{F_{1}}) + C_{2,1} + \\ Cost(\mathcal{L}_{E_{3}}, \mathcal{L}_{F_{2}}) + C_{1,1} + \\ Cost(\mathcal{L}_{E_{4}}, \mathcal{L}_{F_{3}}) + C_{1,1} + \\ Cost(\mathcal{L}_{E_{5}}, \mathcal{L}_{F_{4}} + \mathcal{L}_{F_{5}}) + C_{1,2} + \\ Cost(\mathcal{L}_{E_{6}}, \mathcal{L}_{F_{6}}) + C_{1,1}$$

Find distribution of sentence breaks with minimum cost using dynamic programming



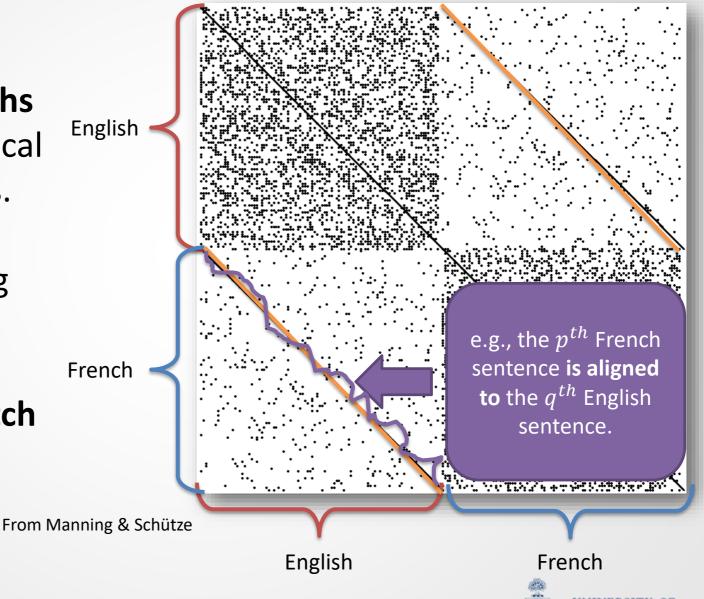
#### 2a. Church's method

 Church (1993) tracks all 4-graphs which are identical across two texts.

English

French

 Each point along this path is considered to represent a match between languages.



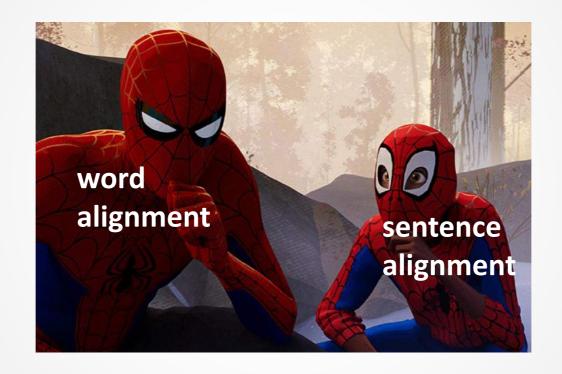
#### 2b. Melamed's method

- LCS(A, B) is the **longest common subsequence** of characters (with gaps allowed) in words A and B.
- Melamed (1993) measures similarity of words A and B  $LCSR(A,B) = \frac{length(LCS(A,B))}{\max(length(A), length(B))}$ 
  - e.g.,

$$LCSR(government, gouvernement) = \frac{10}{12}$$

'LCS Ratio'







#### **Word alignment**

• Word alignments can be 1:1, N:1, 1:N, 0:1,1:0,... E.g.,

"zero fertility" word: not translated (1:0)

Canada 's program has been implemented

Le programme du Canada a été mis en application

"spurious" words: generated from 'nothing' (0:1)

**Note** that this is only one *possible* alignment

One word translated as several words (1:N)

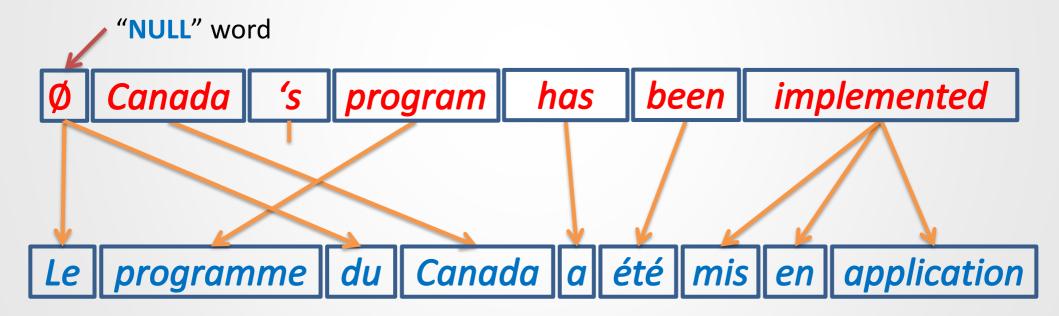


#### **IBM Model 1**



#### **IBM Model 1: the NULL word**

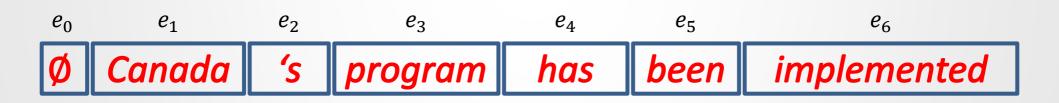
 The NULL word is an imaginary word that we need to account for the production of spurious words.

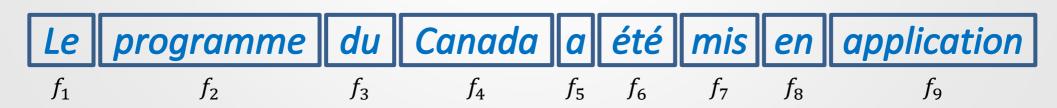




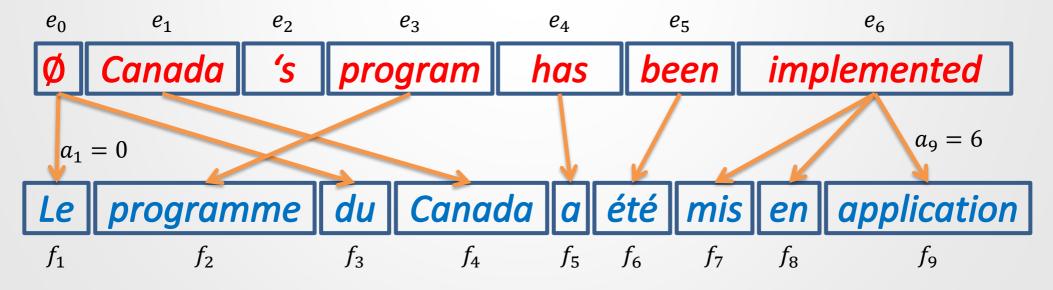
#### **IBM Model 1: some definitions**

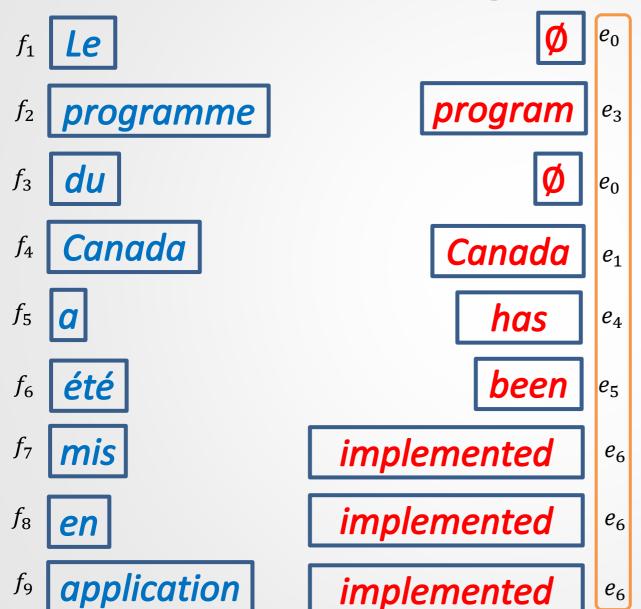
- English sentence E has  $L_E$  words,  $e_1$  ...  $e_{L_E}$ , plus NULL word,  $e_0$ .
- French sentence F has  $L_F$  words,  $f_1 \dots f_{L_F}$ .





- An alignment, a, identifies the English word that 'produced' the given French word at each index.
  - $a = \{a_1, ..., a_{L_F}\}$  where  $a_j \in \{0, ..., L_E\}$
  - E.g.,  $a = \{0, 3, 0, 1, 4, 5, 6, 6, 6\}$

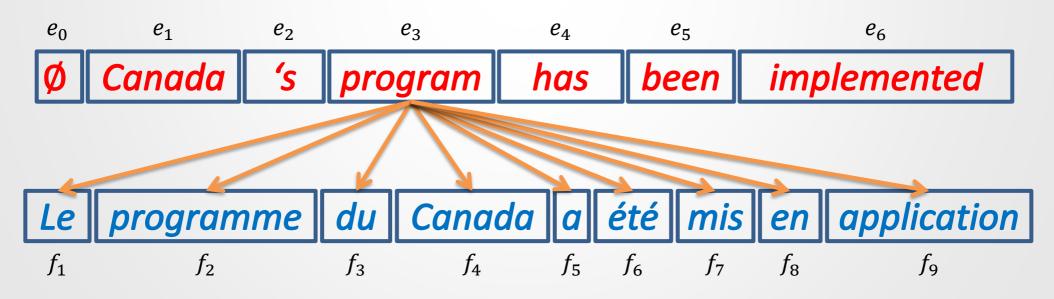




 $a = \{0, 3, 0, 1, 4, 5, 6, 6, 6\}$ 



- There are  $(L_E + 1)^{L_F}$  possible alignments. (since  $||a|| = L_F$ )
- IBM-1 doesn't know that some are very bad in reality.
  - E.g.,  $a = \{3, 3, 3, 3, 3, 3, 3, 3, 3, 3\}$



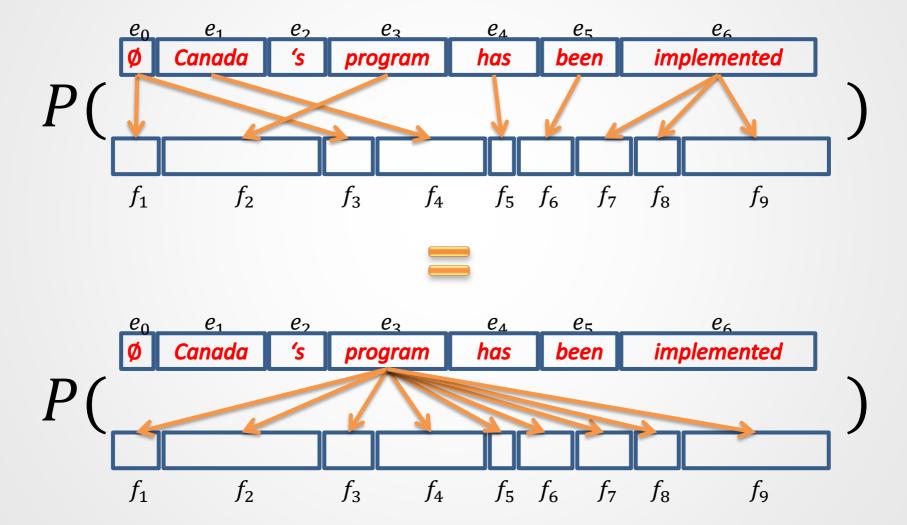
IBM Model 1 assumes that all alignments of E are
 equally likely given only the length (not the words) of F.

$$\forall a, P(a|E, L_F) = \frac{1}{(L_E + 1)^{L_F}}$$

Uniform over all possible alignments.

 This is a major simplifying assumption, but it gets the process started.

# Equally likely alignments a priori





## **IBM Model 1: translation probability**

• Given an alignment  $\alpha$  and an English sentence E, what is the probability of a French sentence F?

• In IBM-1,

$$P(F|a,E) = \prod_{j=1}^{L_F} P(f_j|e_{a_j})$$

(another simplifying assumption)



The probability of the  $j^{th}$ 

French word, given that it

was generated from the

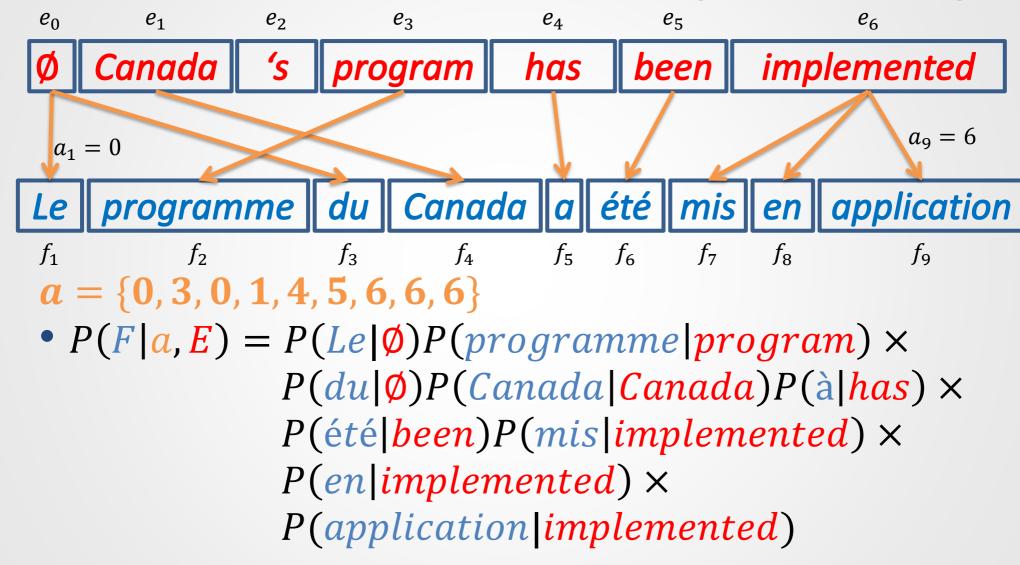
 $a_i^{th}$  English word.

## **IBM Model 1: translation probability**

- E = Canada 's program has been implemented
- $a = \{0,3,0,1,4,5,6,6,6\}$
- F = Le programme du Canada à été mis en application

•  $P(F|a, E) = P(Le|\emptyset)P(programme|program) \times P(du|\emptyset)P(Canada|Canada)P(\grave{a}|has) \times P(\acute{e}t\acute{e}|been)P(mis|implemented) \times P(en|implemented) \times P(application|implemented)$ 

#### **IBM Model 1: translation probability**



## **IBM Model 1: generation**

This is how we imagine English gets corrupted in the noisy channel.

- To generate a French sentence F from English E,
  - 1. Pick a length of F (with probability  $P(L_F)$ ).
  - 2. Pick an alignment (with **uniform** probability,  $\frac{1}{(L_E+1)^{L_F}}$ ).
  - 3. Sample French words with probability

Slide 17

$$P(F|a,E) = \prod_{j=1}^{L_F} P(f_j|e_{a_j})$$
 Slide 19

So,  

$$P(F, a|E) = P(a|E)P(F|a, E) = \frac{P(L_F)}{(L_E + 1)^{L_F}} \prod_{j=1}^{L_F} P(f_j|e_{a_j})$$



#### IBM-1: alignment as hidden variable

• If P(F, a|E) describes the process of generating French words and alignments from English words...

Then

Remember, the **noisy channel model** states that French words are really encoded English words!

$$P(F|E) = \sum_{a \in A} P(F, a|E)$$

where  $\mathcal{A}$  is the **set** of **all possible** alignments



#### **IBM-1: training**

- Our training data  $\mathcal{O}$  is a set of pairs of corresponding French and English sentences,  $\mathcal{O} = \{(F_i, E_i)\}, i = 0...N$ .
- If we knew the word alignments, a, learning P(f|e) would be trivial with MLE:  $P(f|e) = \frac{Count(f,e)}{Count(e)}$ .
- But the alignments are hidden. We need to use ...





#### **IBM-1: expectation-maximization**



- 1. Initialize translation parameters P(f|e) (e.g., randomly).
- **2. Expectation**: Given the current  $\theta_k = P(f|e)$ , compute the **expected value** of Count(f,e) for all words in training data O.
- 3. Maximization: Given the expected value of Count(f, e), compute the maximum likelihood estimate of  $\theta_k = P(f|e)$

## **IBM-1 EM: Example**

Imagine our training data is

```
O = {(blue house, maison bleue),
    (the house, la maison)}
```

The vocabularies are

```
\mathcal{V}_E = \{blue, house, the\} and \mathcal{V}_F = \{maison, bleue, la\}.
```

 For simplicity, we consider only 1:1 alignments: there is no NULL word, there are no zero-fertility words.



#### **IBM-1 EM: Example**

- First, we **initialize** our parameters,  $\theta = P(f|e)$ .
- In the Expectation step, we compute expected counts:
  - TCount(f, e): the total number of times e and f are aligned.
  - Total(e): the total number of e.

    This has to be done in steps by first computing P(F, a|E) then P(a|F, E)
- In the Maximization step, we perform MLE with the expected counts.



#### **IBM-1 EM: Example initialization**

1. Make a table of P(f|e) for all possible pairs f and e.

Initialize uniformly across rows.

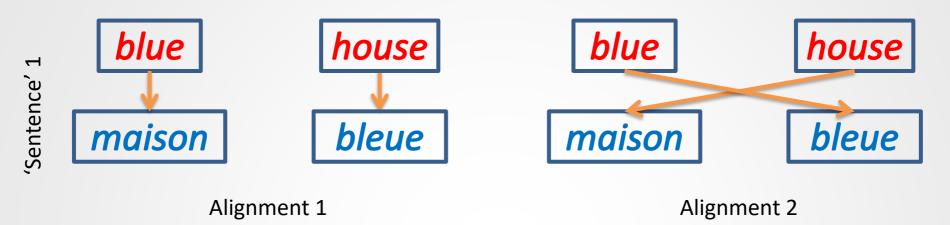
 $\theta_0$ :

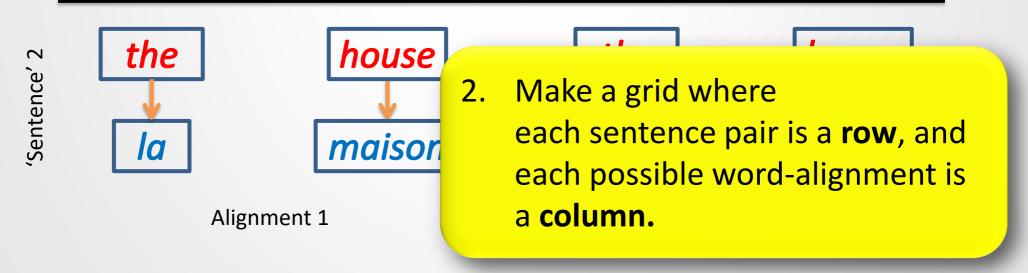
$$P(maison|blue) = \frac{1}{3} \qquad P(bleue|blue) = \frac{1}{3} \qquad P(la|blue) = \frac{1}{3}$$

$$P(maison|house) = \frac{1}{3} \qquad P(bleue|house) = \frac{1}{3} \qquad P(la|house) = \frac{1}{3}$$

$$P(maison|the) = \frac{1}{3} \qquad P(bleue|the) = \frac{1}{3} \qquad P(la|the) = \frac{1}{3}$$

## IBM-1 E: compute P(F|a, E)





# IBM-1 E: compute P(F|a, E)

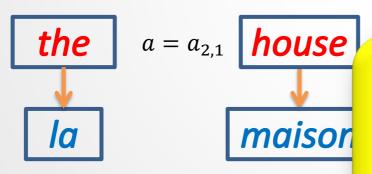
blue  $a = a_{1,1}$  house maison bleue

$$P(F|a, E) = P(maison|blue) \times P(bleue|house) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$$

blue  $a = a_{1,2}$  house

bleue

 $P(F|a, E) = P(bleue|blue) \times P(maison|house) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$ 



$$P(F|a, E) = P(la|the) \times$$

$$P(maison|house) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$$

3. For each sentence pair and alignment, compute (slide 19)

maison

$$P(F|a,E) = \prod_{f_j} P(f_j|e_{a_j})$$

• We want the **probability of an alignment**  $\alpha$  so that we can compute the **expected**  $Count(f_i, e_i)$ .

$$P(a|E,F) = \frac{P(F|a,E)}{\sum_{a_i \in \mathcal{A}} P(F|a_i,E)}$$

- This is **not** the same as the probability  $P(a|E, L_F)$ .
  - i.e., it won't always be uniform.

$$P(a|E,F) = \frac{P(a,E,F)}{P(E,F)} = \frac{P(a,E,F)}{P(E)P(F|E)} = \frac{P(F,a|E)P(E)}{P(E)P(F|E)}$$

$$= \frac{P(F,a|E)}{\sum_{a_i \in \mathcal{A}} P(F,a_i|E)} = \frac{P(F|a,E)}{\sum_{a_i \in \mathcal{A}} P(F|a_i,E)}$$

(\*) Because 
$$P(F|E) = \sum_{a_i \in \mathcal{A}} P(F, a_i|E)$$
 (slide 23)

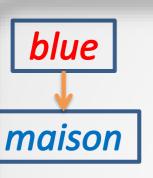
(\*\*) Rewrite 
$$P(F, a|E)$$
 as on slide 22, and  $\frac{P(L_F)}{(L_E+1)^{L_F}}$  cancels out

• We want the **probability of an alignment** a so that we can compute the **expected**  $Count(f_i, e_i)$ .

$$P(a|E,F) = \frac{P(F|a,E)}{\sum_{a_i \in \mathcal{A}} P(F|a_i,E)}$$

- This is **not** the same as the probability  $P(a|E, L_F)$ .
  - i.e., it won't always be uniform.

Sentence' 1



house

bleue

$$P(a|E,F) = \frac{1/9}{1/9 + 1/9} = \frac{1}{2}$$

blue

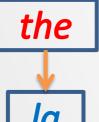
house

maison

bleue

$$P(a|E,F) = \frac{1/9}{1/9 + 1/9} = \frac{1}{2}$$

Sentence' 2



house

maisor

$$P(a|E,F) = \frac{1/9}{1/9 + 1/9} = \frac{1}{2}$$

4. For each element in your grid, divide P(F|a,E) by the sum of the row (slide 33).

## IBM-1 E: compute *TCount*

maison and blue are aligned only in alignment 1, sentence 1.

$$P(a = 1|F_1, E_1) = \frac{1}{2}$$

TCount(maison, blue) TCount(bleue, l

$$=\frac{1}{2}$$

TCount(maison, house) TCount(bleue, house)

$$= \frac{1}{2} + \frac{1}{2} = 1$$

TCount(maison, the)

=?

This is a **new** table, **not** the  $\theta = P(f|e)$  table from before!

maison and house are aligned in alignment 2, sentence 1. and alignment 1, sentence 2

$$P(a = 2|F_1, E_1) = \frac{1}{2}$$

$$P(a = 1|F_2, E_2) = \frac{1}{2}$$

**5.** For each possible word pair e and f, **sum** P(a|E,F) from step 4 across all alignments and sentence pairs for each instance that *e* is aligned with *f* 



## IBM-1 <u>E</u>: compute *TCount*

$TCount(maison, blue) = \frac{1}{2}$	$TCount(bleue, blue) = \frac{1}{2}$	TCount(la, blue) = 0
$TCount(maison, house)$ $= \frac{1}{2} + \frac{1}{2} = 1$	$TCount(bleue, house) = \frac{1}{2}$	$TCount(la, house) = \frac{1}{2}$
$TCount(maison, the)$ $= \frac{1}{2}$	TCount(bleue, the) = 0	$TCount(la, the) = \frac{1}{2}$

## IBM-1 E: compute *Total*

E.g., 
$$Total(blue) = \frac{1}{2} + \frac{1}{2} = 1$$
,  $Total(house) = 1 + \frac{1}{2} + \frac{1}{2} = 2$ , ...

$$TCount(maison, blue)$$
  $TCount(bleue, blue)$   $TCount(la, blue)$   $= \frac{1}{2}$   $= 0$ 

TCount(maison, house) TCount(bleue, house)

$$=\frac{1}{2}+\frac{1}{2}=1$$

*TCount*(maison, the)

$$=\frac{1}{2}$$

$$=\frac{1}{2}$$

TCount(la, house)

$$=\frac{1}{2}$$

6. Sum over the rows of this table to get the **total** estimates for **each** English word, e.

# IBM-1 M: Recompute P(f|e)

7. Compute 
$$P(f|e) = \frac{TCount(f,e)}{Total(e)}$$
  
This is your model after iteration 1.

 $\theta_1$ :

P(maison bl)	ue)	P(la blue)
	1/2   1/2	0
= -	$\frac{}{1}$ = $\frac{}{1}$	$=\frac{1}{1}$
P(maison hou	rse) P(bleue house)	P(la house)
	1/2 $1/2$	1/2 1
:	$=\frac{1}{2}$ $=\frac{1}{2}$ $=\frac{1}{4}$	$=\frac{7}{2}=\frac{7}{4}$
P(maison t	P(bleue the)	P(la the)
	1/2	1/2
= -	$\frac{}{1}$ = $\frac{}{1}$	=

### **IBM-1 EM: Repeat**

- You have finished 1 iteration of EM when you have completed Step 7,
- Go back to Step 2 and repeat.



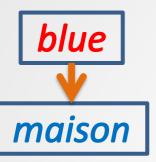
### **IBM-1 EM: Repeat**

- 1. Initialize P(f|e)
- 2. Make grid of all possible alignments
- 3. Compute  $P(F|a, E) \rightarrow Products of P(f|e)$
- 4. Compute  $P(a|E,F) \rightarrow$  Divide by sum of rows from step 3
- Compute *TCount* → Sum relevant probabilities from step 4
- 6. Compute  $Total \rightarrow$  Sum over rows from step 5
- 7. Compute  $P(f|e) = \frac{TCount(f,e)}{Total(e)}$



## IBM-1 E: compute P(F|a, E)

Sentence' 1





bleue

blue

house

maison

bleue

$$P(F|a, E) = P(maison|blue) \times$$

$$P(bleue|house) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

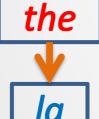
 $P(F|a, E) = P(bleue|blue) \times$ 

$$P(maison|house) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

2: make grid

3: compute products of P(f|e)

Sentence' 2



house



the

house

la

maison

$$P(F|a, E) = P(la|the) \times$$

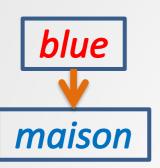
$$P(maison|house) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

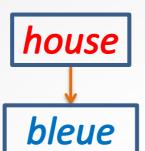
$$P(F|a, E) = P(maison|the) \times$$

$$P(la|house) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

## IBM-1 E: compute P(a|E,F)

Sentence' 1





$$P(a|E,F) = \frac{1/8}{1/8 + 1/4} = \frac{1}{3}$$

blue

house

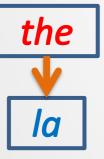
maison

bleue

$$P(a|E,F) = \frac{1/4}{1/8 + 1/4} = \frac{2}{3}$$

4: divide by sum of rows in step 3

Sentence' 2





$$P(a|E,F) = \frac{1/4}{1/4 + 1/8} = \frac{2}{3}$$

the

house

la

maison

$$P(a|E,F) = \frac{1/8}{1/4 + 1/8} = \frac{1}{3}$$

### IBM-1 E: compute *TCount* & *Total*

$$Total(blue) = \frac{1}{3} + \frac{2}{3} = 1, \quad Total(house) = \frac{4}{3} + \frac{1}{3} + \frac{1}{3} = 2,$$
$$Total(the) = \frac{1}{3} + \frac{2}{3} = 1$$
5. Compute *TCount* by summing relevant probabilities from step 4

Total (the)  $-\frac{1}{3} + \frac{1}{3}$ relevant probabilities from step 4
6. Compute Total by summing rows

TCount(maison, blue)

TCount(bleue, blue)  $\frac{1}{3} + \frac{1}{3} - \frac{1}{3}$ TCount(bleue, blue)  $\frac{1}{3} + \frac{1}{3} - \frac{1}{3}$ TCount(bleue, blue)  $\frac{1}{3} + \frac{1}{3} - \frac{1}{3}$ TCount(la, blue)  $\frac{1}{3} + \frac{1}{3} - \frac{1}{3}$ TCount(bleue, blue)  $\frac{1}{3} + \frac{1}{3} - \frac{1}{3}$ TCount(la, blue)

 $= \frac{1}{3}$   $= \frac{1}{3}$   $= \frac{2}{3}$   $= \frac{1}{3}$   $= \frac{1}{3}$ 

# IBM-1 M: Recompute P(f|e)

• Compute  $P(f|e) = \frac{TCount(f,e)}{Total(e)}$  $\theta_2$ : Ties have been broken

e.g.,

P(maison|blue)

 $\neq P(bleue|blue)$ 

$P(maison blue) = \frac{1/3}{1}$	$P(bleue blue) = \frac{2/3}{1}$	$P(la blue) = \frac{0}{1}$
$P(maison house) = \frac{4/3}{2} = \frac{2}{3}$	$P(bleue house) = \frac{1/3}{2} = \frac{1}{6}$	$P(la house) = \frac{1/3}{2} = \frac{1}{6}$
$P(maison the) = \frac{1/3}{1}$	$P(bleue the) = \frac{0}{1}$	$P(la the) = \frac{2/3}{1}$

### Practical note on programming IBM-1

- If you were to code the EM algorithm for IBM-1, you would **not** initialize  $\theta = P(f|e)$  uniformly over the **entire** vocabulary.
  - Don't make a  $V_F \times V_E$  table with  $P(f|e) = 1/||V_E||$



- This structure would be too large.
  - Probabilities would be too small.
  - It would take too much work to update.
- Rather, initialize a **hash table** over **possible** alignments,  $\mathcal{M}$ . For every English word e, only consider French words f in sentences aligned with English sentences containing e.
  - e.g., structure P. e.  $f := P(f|e) = 1/||\mathcal{M}||$



### **Higher IBM models**

IBM Model 1	lexical translation	
IBM Model 2	adds absolute <b>re-ordering model</b>	
IBM Model 3	adds fertility model	
•••	•••	

- Only IBM Model 1 training reaches a global maximum
  - Training of each IBM model extends the next lowest model.
- Higher models become computationally expensive.



#### IBM-2

- Unlike IBM Model-1, the placement of a word in, say, Spanish in IBM Model-2 depends on where its equivalent word was in English.
  - IBM-2 captures the intuition that translations should lie roughly "along the diagonal".

	Buenos	dias	,	me	gusta	papas	frías
Good	X						
day		Χ					
,			X				
1				Χ			
like					X		
cold							Х
potatoes						X	



#### IBM-2

 IBM Model 2 builds on Model 1 by adding a re-ordering model defined by distortion parameters <u>regardless of actual words</u>.

$$D(i|j,\mathcal{L}_E,\mathcal{L}_F)$$
= the probability that the  $i^{th}$  English slot is aligned to the  $j^{th}$  French slot, given sentence lengths  $\mathcal{L}_E$  and  $\mathcal{L}_F$ .

In IBM Model 2:

$$P(a|E,\mathcal{L}_{E},\mathcal{L}_{F}) = \prod_{j=1}^{\mathcal{L}_{F}} D(a_{j}|j,\mathcal{L}_{E},\mathcal{L}_{F})$$

Recall that in IBM Model 1,

$$P(\boldsymbol{a}|\boldsymbol{E}, \boldsymbol{\mathcal{L}}_{\boldsymbol{E}}, \boldsymbol{\mathcal{L}}_{\boldsymbol{F}}) = \frac{P(\boldsymbol{\mathcal{L}}_{\boldsymbol{F}})}{(\boldsymbol{\mathcal{L}}_{\boldsymbol{E}} + 1)^{\boldsymbol{\mathcal{L}}_{\boldsymbol{F}}}}$$



### IBM-2 - Probability of alignment

- $\bullet$  E = And the program has been implemented
- F = Le programme a été mis en application
- $\mathcal{L}_E = 6$
- $\mathcal{L}_F = 7$
- $a = \{2,3,4,5,6,6,6\}$  (i.e.,  $f_1 \leftarrow e_2, f_2 \leftarrow e_3,...$ )

 $D(2^{\text{nd}} \text{ English word} | 1^{\text{st}} \text{ French word,...})$ 

• 
$$P(a|E, \mathcal{L}_E, \mathcal{L}_F) = D(2|1,6,7) \times D(3|2,6,7) \times D(4|3,6,7) \times D(5|4,6,7) \times D(5|4,6,7) \times D(6|5,6,7) \times D(6|6,6,7) \times D(6|7,6,7)$$

This is independent of the actual words.

This cares only about position.



### **IBM-2:** generation

- To generate a French sentence F from English E,
  - 1. Pick an alignment with probability

$$\prod_{j=1}^{\mathcal{L}_F} D(a_j|j,\mathcal{L}_E,\mathcal{L}_F)$$

3. Sample French words with probability

$$P(F|a,E) = \prod_{j=1}^{\mathcal{L}_F} P(f_j|e_{a_j})$$
 This is the same  $P(f|e)$  as in IBM-1.

as in IBM-1.

So,
$$P(F, a|E) = P(a|E)P(F|a, E) = \prod_{j=1}^{\mathcal{L}_F} D(a_j|j, \mathcal{L}_E, \mathcal{L}_F)P(f_j|e_{a_j})$$

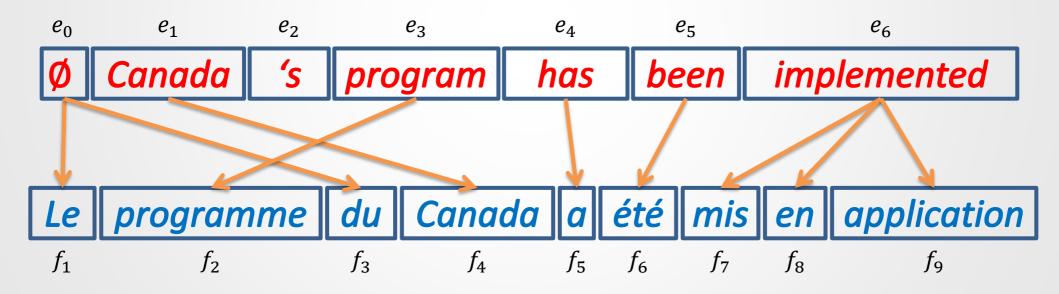


### **IBM-2: training**

- We use EM, as before with IBM-1 except that we need to take the distortion into account when computing the probability of an alignment.
- We also need to learn the distortion function.
- Aren't you glad that you don't need to know how to compute EM for IBM-2?

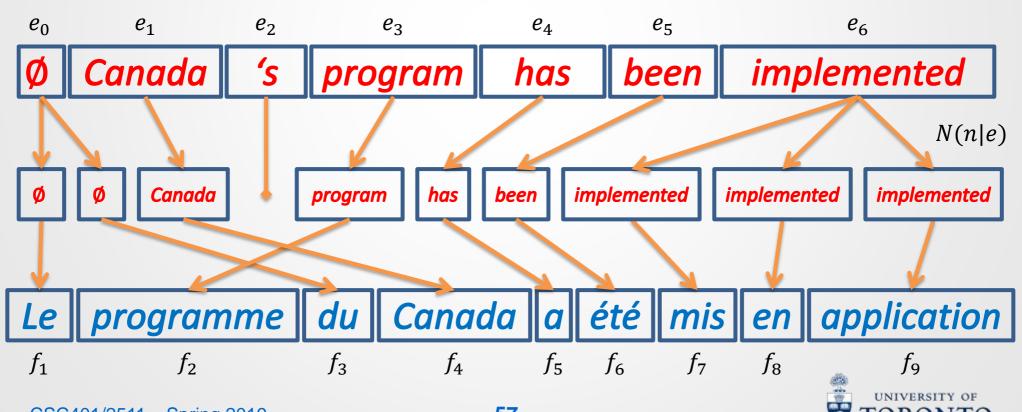
#### IBM-3

- **IBM Model 3** extends Model 2 by adding a **fertility model** that describes how many French words each **English** word can produce.
  - In the example below, implemented appears to be more fertile than program.



### **IBM-3: The generation model**

- First, we **replicate** each word according to a new hidden parameter, N(n|e), which is the **probability that word** e **produces** n **words**.
  - We then re-align (with distortion) and translate as we did in IBM-2.



### **IBM** models

IBM Model 1	lexical translation	
IBM Model 2	adds absolute <b>re-ordering model</b>	
IBM Model 3	adds fertility model	



### Reading

- Entirely optional: Vogel, S., Ney, H., and Tillman, C. (1996). *HMM-based Word Alignment in Statistical Translation*. In: Proceedings of the 16th International Conference on Computational Linguistics, pp. 836-841, Copenhagen.
- (optional) Gale & Church "A Program for Aligning Sentences in Bilingual Corpora" (on course website)
- Useful reading on IBM Model-1: Section 25.5 of the 2<sup>nd</sup> edition of the Jurafsky & Martin text.
  - 1<sup>st</sup> edition available at Robarts library.
- Other: Manning & Schütze Sections 13.0, 13.1.2
   (Gale&Church), 13.1.3 (Church), 13.2, 13.3, 14.2.2

