

# Computational Linguistics

CSC 2501 / 485  
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3

## 3. Chart parsing

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Reading: Jurafsky & Martin: 13.3–4.

Allen: 3.4, 3.6.

Bird et al.: 8.4, online extras 8.2 to end of  
section “Chart Parsing in NLTK”.

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# Efficient parsing

- Want to avoid problems of blind search:
  - Avoid redoing analyses that are identical in more than one path of the search.
- Guide the analysis with both
  - the actual input
  - the expectations that follow from the choice of a grammar rule.
- Combine strengths of both top-down and bottom-up methods.

# Chart parsing

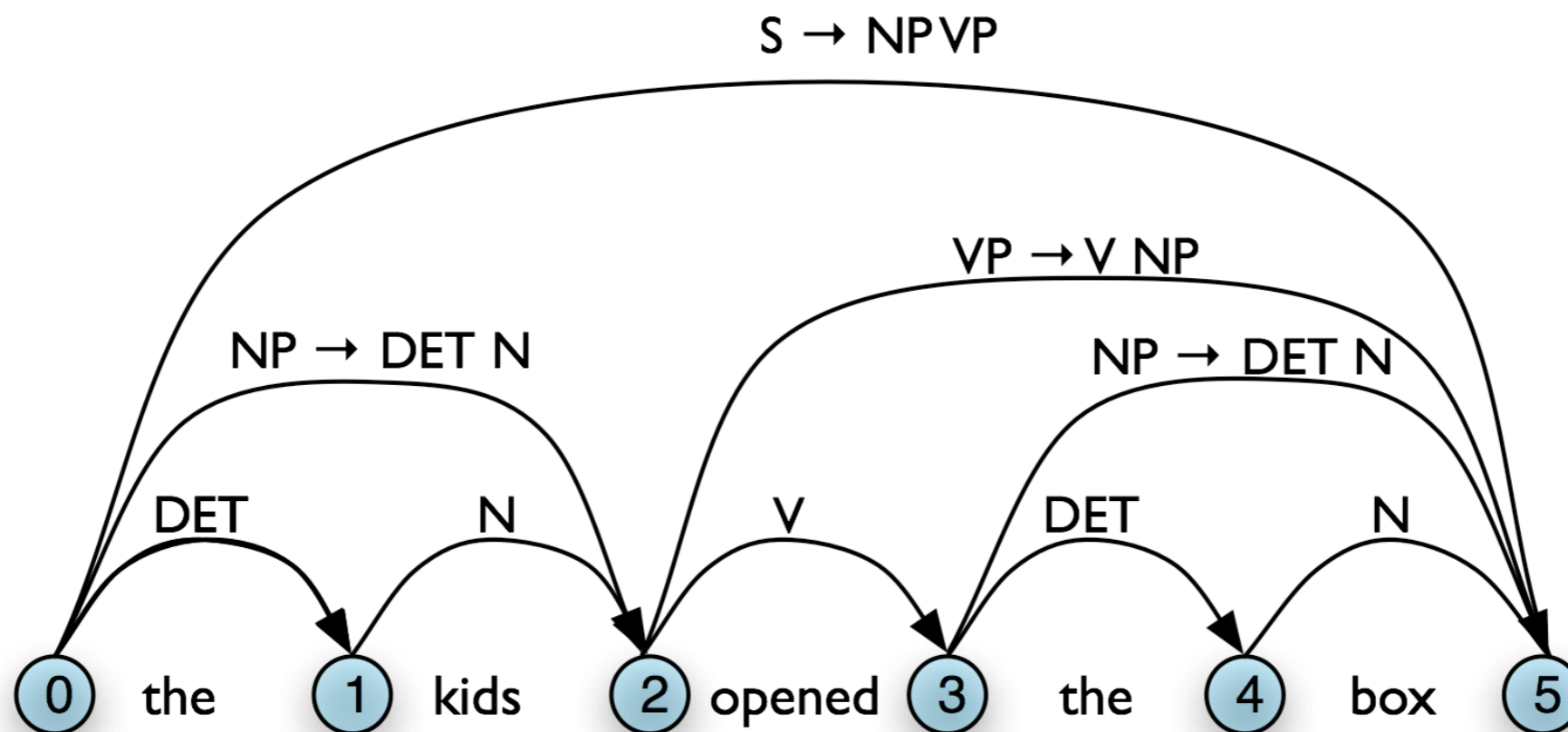
- Main idea:
  - Use data structures to maintain information:  
a **chart** and an **agenda**
- **'Agenda':**
  - List of constituents that need to be processed.
- **'Chart':**
  - Records (“*memorizes*”) work; obviates repetition.
  - Related to: Well-formed substring table (WFST);  
CKY parsing;  
Earley parsing;  
dynamic programming.

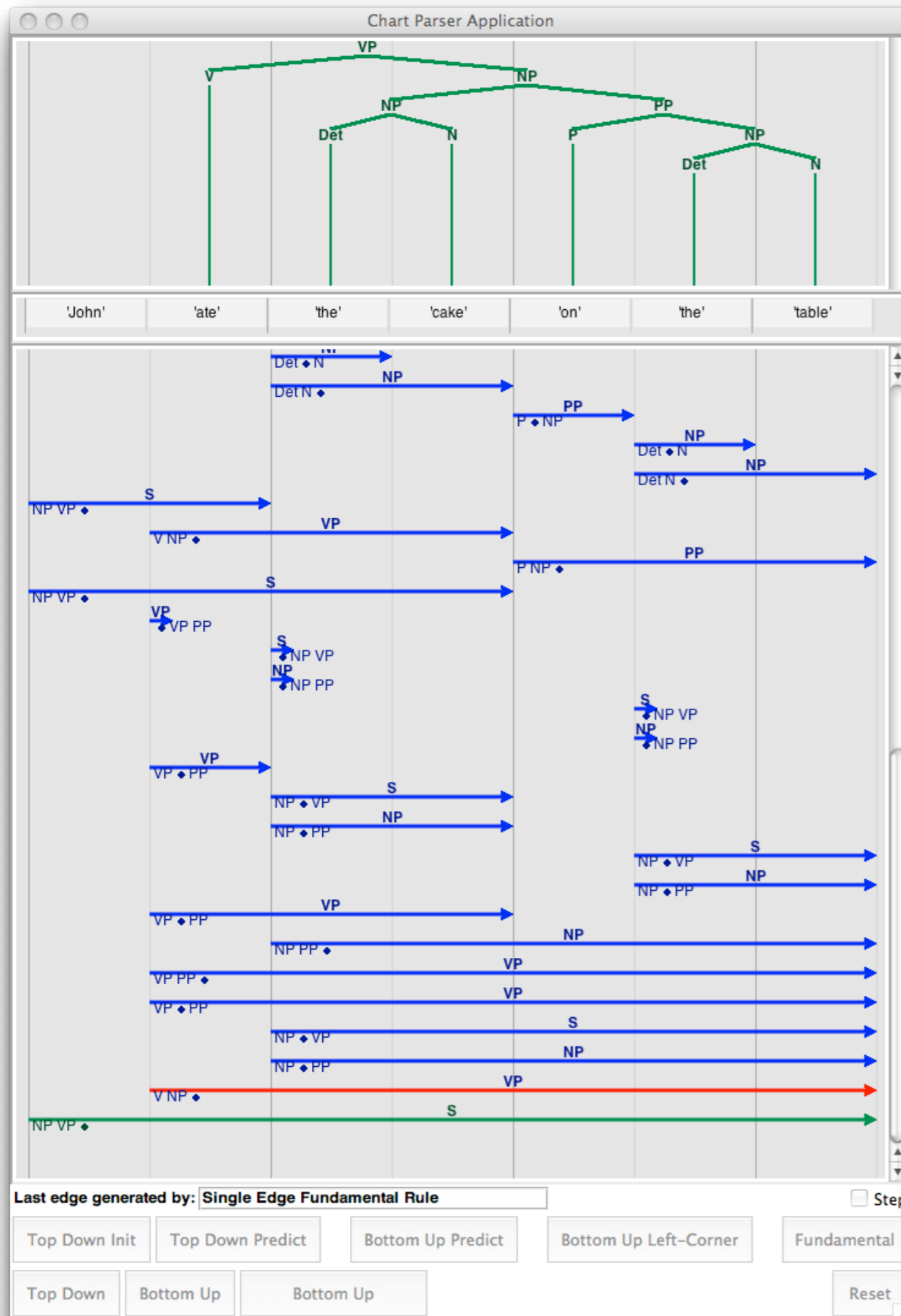
# Charts 1

- Contents of chart:
  1. Partially built constituents (also called **active arcs**).  
Think of them as *hypotheses*.
  2. Completed constituents (**inactive arcs**).
- Representation: Labelled arc (**edge**) from one point in sentence to another (or same point).
  - Directed; always left-to-right (or to self).
  - Label is grammar rule used for arc.

# Charts 2

- Notation for positions in sentence from 0 to  $n$  (length of sentence):
- $_0$  *The*  $_1$  *kids*  $_2$  *opened*  $_3$  *the*  $_4$  *box*  $_5$

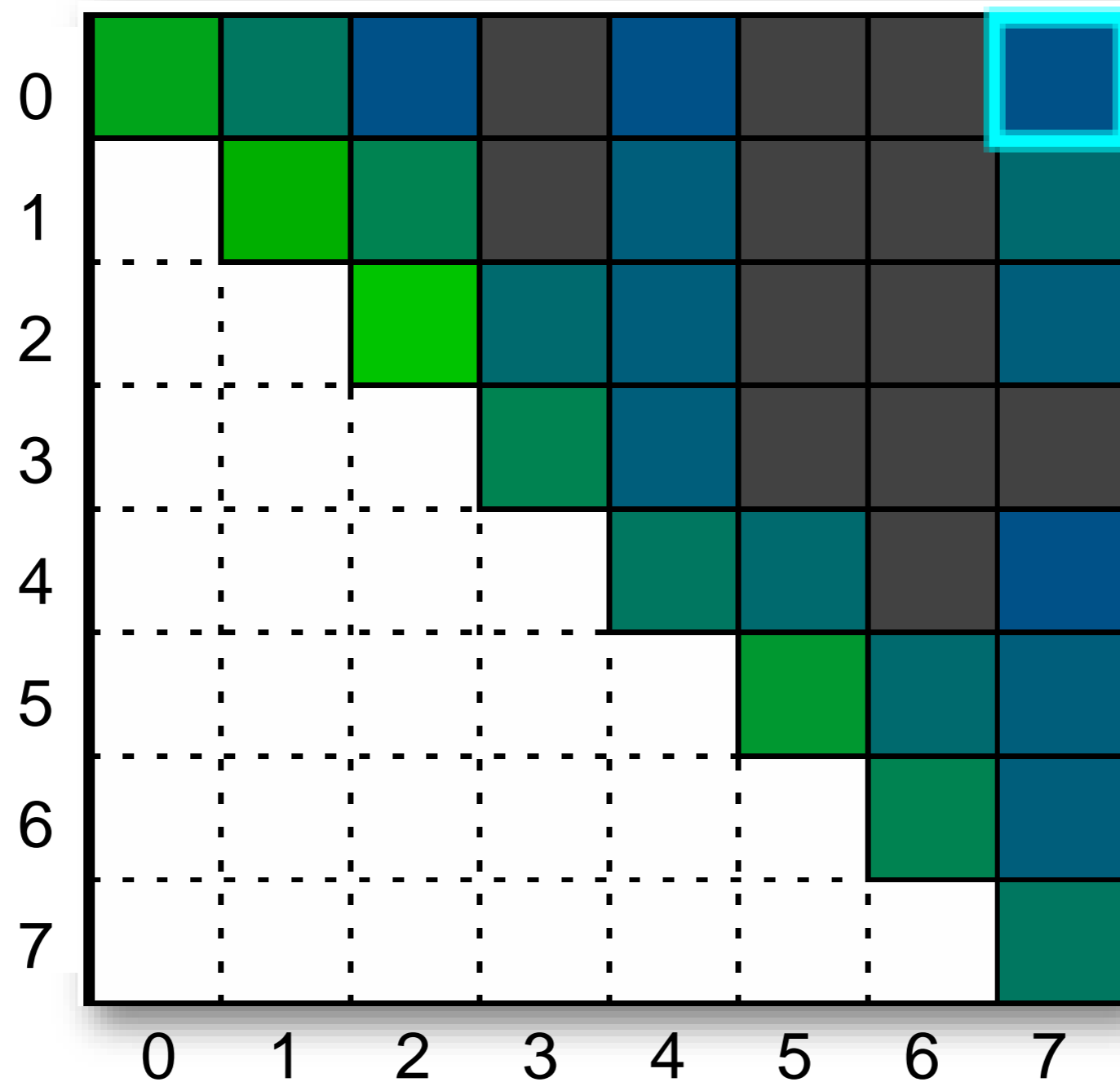




Part of a chart from the NLTK  
chart parser demo,  
`nltk.app.chartparser()`

# Charts 3

- An arc can connect any positions  $i, j$  ( $0 \leq i \leq j \leq n$ ).
- You can have  $> 1$  arc on any  $i, j$ .
- You can associate all arcs on positions  $i, j$  with cell  $ij$  of upper-triangular matrix.



Arcs in top right corner cell cover the whole sentence.

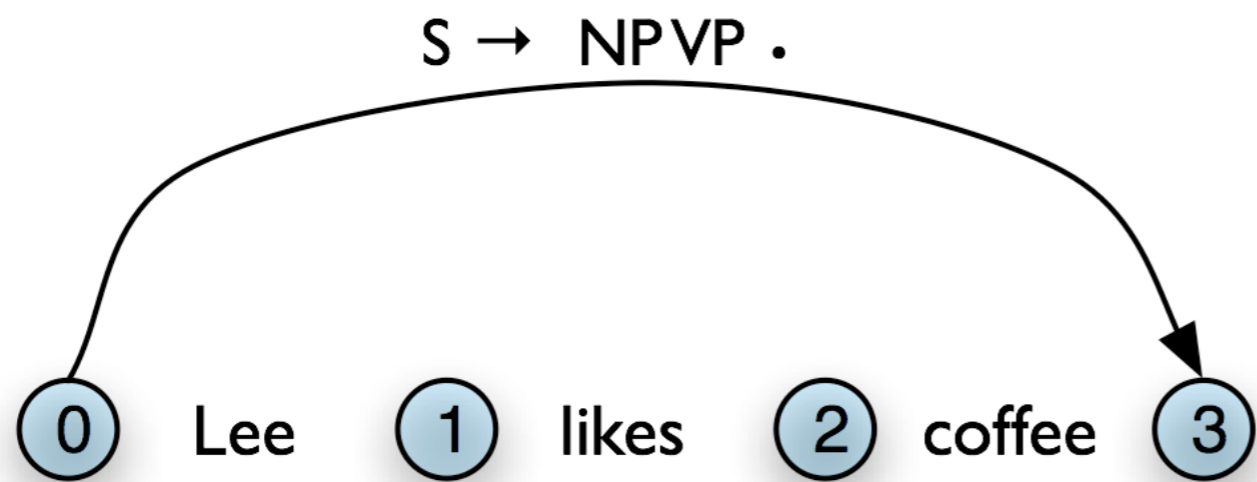
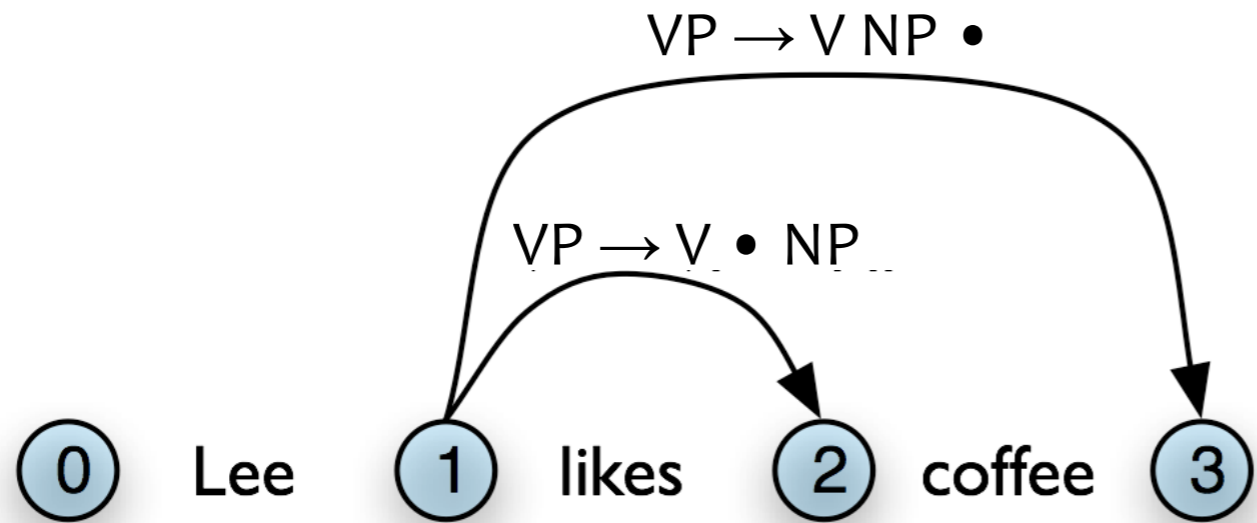
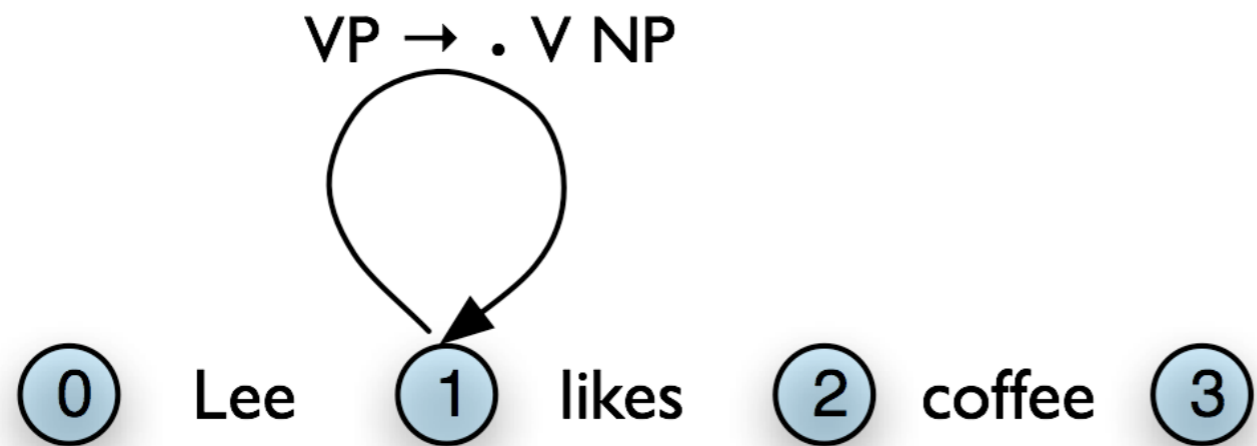
Those for S are 'parse edges'.

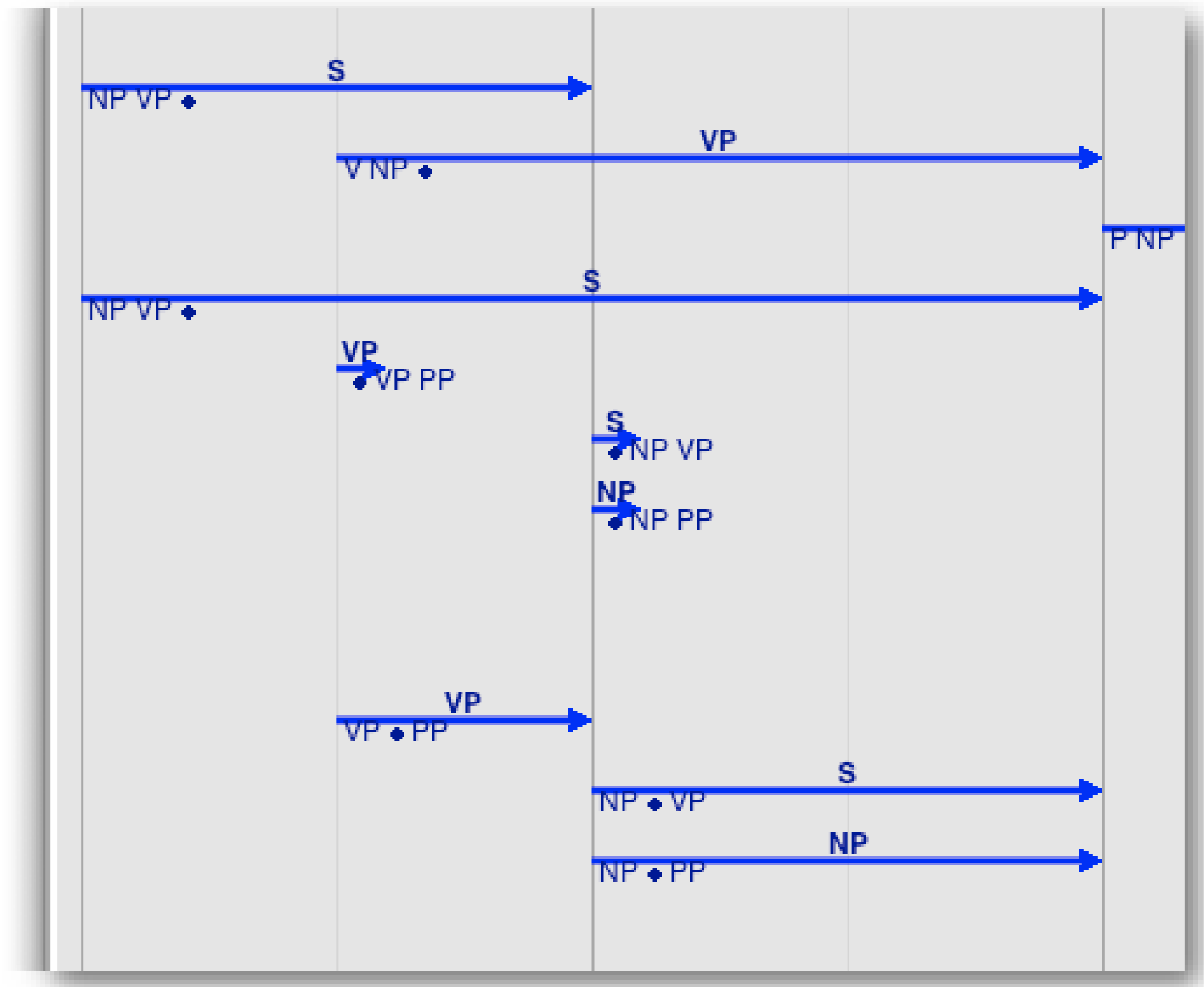
The matrix for a seven-word sentence from the NLTK chart parser demo  
`nltk.app.chartparser()`



# Notation for arc labels

- *Notation*: ‘•’ means ‘complete to here’.
  - $A \rightarrow X Y \bullet Z$  (*active*)  
‘In parsing an  $A$ , we’ve so far seen an  $X$  and a  $Y$ , and our  $A$  will be complete once we’ve seen a  $Z$ .’
  - $A \rightarrow X Y Z \bullet$  (*inactive*)  
‘We have seen an  $X$ , a  $Y$ , and a  $Z$ , and hence completed the parse of an  $A$ .’
  - $A \rightarrow \bullet X Y Z$  (*active*)  
‘In parsing an  $A$ , so far we haven’t seen anything.’





Part of a chart from the NLTK chart parser demo,  
`nltk.app.chartparser()`

# Fundamental rule of chart parsing

- **Arc extension:**

Let  $X$ ,  $Y$ , and  $Z$  be sequences of symbols, where  $X$  and  $Y$  are possibly empty.

If the chart contains an **active** arc from  $i$  to  $j$  of the form

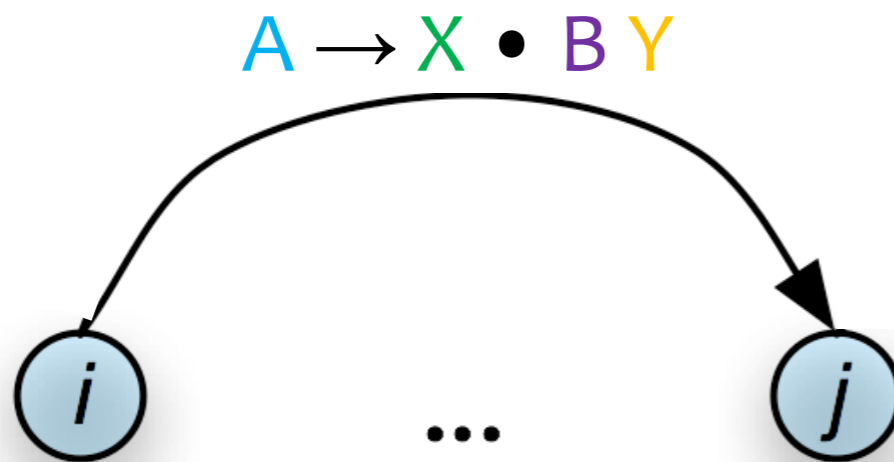
$$A \rightarrow X \bullet B Y$$

and a completed arc from  $j$  to  $k$  of the form

$$B \rightarrow Z \bullet \quad \text{or} \quad B \rightarrow \textit{word}$$

then add an arc from  $i$  to  $k$

$$A \rightarrow X B \bullet Y$$



# Bottom-up arc-addition rule

- **Arc addition (or prediction):**

If the chart contains an completed arc from  $i$  to  $j$  of the form

$$A \rightarrow X \bullet$$

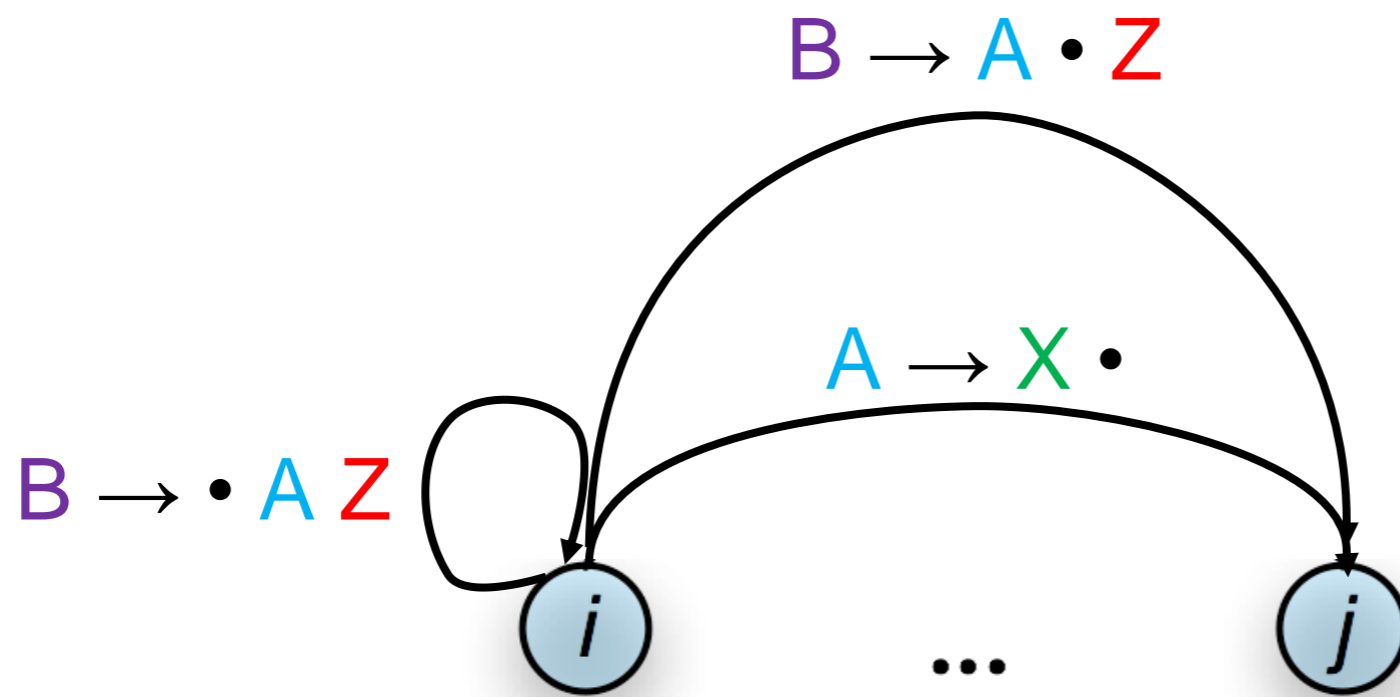
and the grammar contains a rule

$$B \rightarrow A Z$$

then add an arc from  $i$  to  $i$  (*reflexive*)

$$B \rightarrow \bullet A Z$$

or an arc  $B \rightarrow A \bullet Z$  from  $i$  to  $j$ .



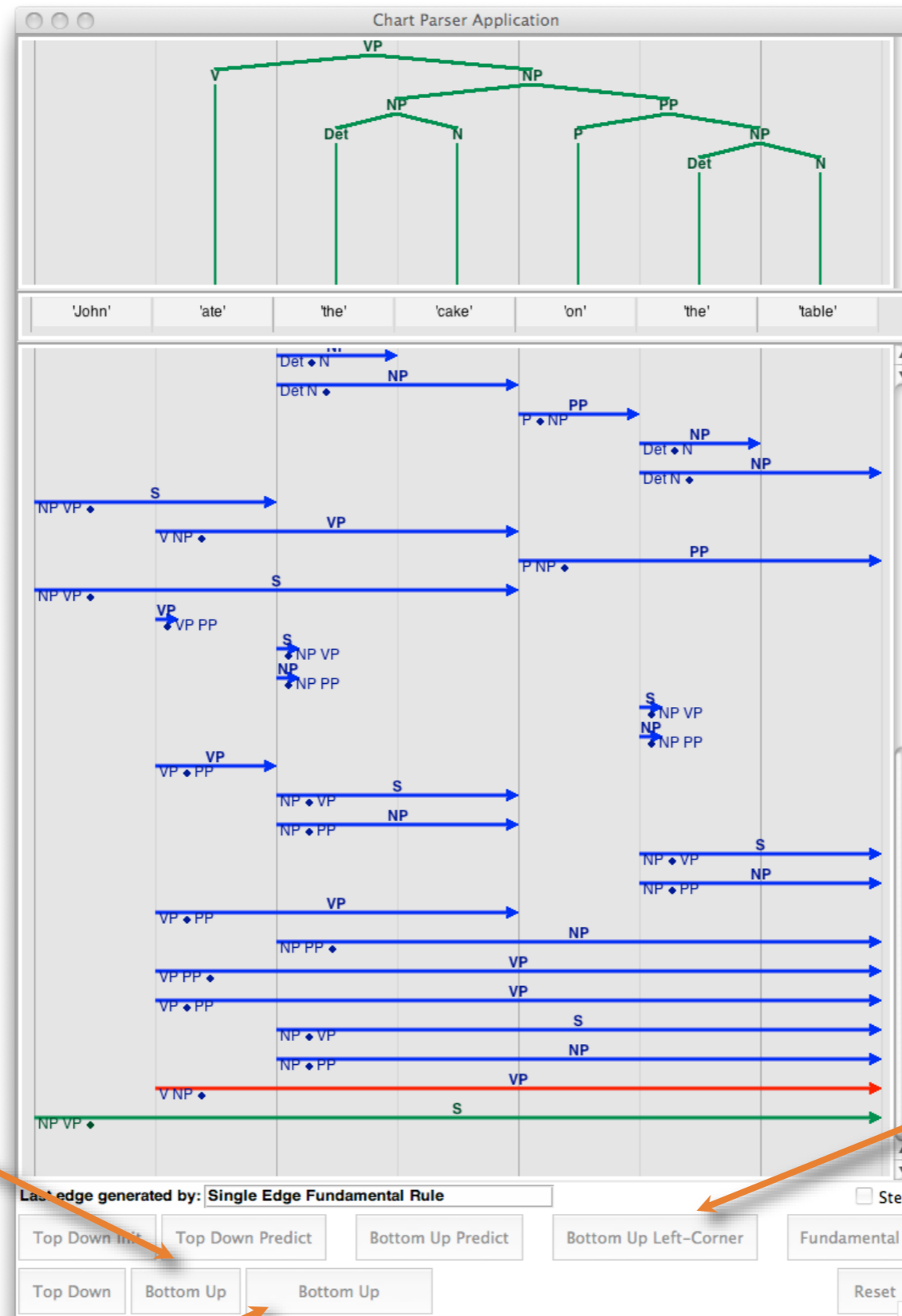
# Bottom-up chart parsing

## BKL's view

- Initialize chart with **each word** in the **input** sentence.
- Until nothing more happens:
  - Apply the **bottom-up addition** rule wherever you can.
  - Apply the **fundamental rule** wherever you can.
- Return the trees corresponding to the parse edges in the chart.



```
>>> nltk.app.chartparser()
```



Top-down Init Rule

Top-down Predict Rule

Top-down Strategy

Bottom-up Strategy

Bottom-up Left-Corner Strategy

Bottom-up Predict Rule

Bottom-up Left-Corner Predict Rule

Fundamental Rule

Reset Parser

# Observations

- This cool thing builds all constituents exactly once.
- It never re-computes the prefix of an RHS.
- It exploits context-free nature of rules to reduce the search. How?

# Controlling the process

- Doing everything you can is too uncontrolled.
- Try to avoid predictions and expansions that will lead nowhere, dummy.
- So use an **agenda** — a list of completed arcs.
  - When an arc is completed, it is initially added to the agenda, not the chart.
  - **Agenda rules** *decide* which completed arc to move to the chart next.
  - *E.g.*, treat agenda as *stack* or as *queue*; or pick item that looks “most efficient” or “most likely”; or pick NPs first; or ....

# Bottom-up chart parsing

## Jurafsky & Martin's view

- Initialize agenda with the list of **lexical categories** (Pos) of each word in the input sentence.
- **Until agenda is empty**, repeat:
  - Move next constituent **C** from agenda to chart.
    - i. Find rules whose RHS starts with **C** and add corresponding active arcs to the chart.
    - ii. Find active arcs that continue with **C** and extend them; add the new active arcs to the chart.
    - iii. Find active arcs that have been completed; add their LHS as a new constituent to the agenda.

# Bottom-up chart parsing

## Algorithm the first

INITIALIZE:

```
set Agenda = list of all possible categories of each input word  
                (in order of input);  
set n = length of input;  
set Chart = ();
```

ITERATE:

```
loop  
    if Agenda = () then  
        if there is at least one S constituent from 0 to n then  
            return SUCCESS 😊  
        else  
            return FAIL 😞  
        end if  
    else ...
```

# Bottom-up chart parsing

## Algorithm the second

**Set**  $C_{i,j} = \text{First}(\text{Agenda});$  */\* Remove first item from agenda. \*/*  
*/\*  $C_{i,j}$  is a completed constituent of type C from position i to position j \*/*  
**Add**  $C_{i,j}$  to Chart;

ARC UPDATE:

a. BOTTOM-UP ARC ADDITION (PREDICTION):

**for each** grammar rule  $X \rightarrow C X_1 \dots X_N$  **do**

Add arc  $X \rightarrow C \bullet X_1 \dots X_N$ , from  $i$  to  $j$ , to Chart;

b. ARC EXTENSION (FUNDAMENTAL RULE):

**for each** arc  $X \rightarrow X_1 \dots \bullet C \dots X_N$ , from  $k$  to  $i$ , **do**

Add arc  $X \rightarrow X_1 \dots C \bullet \dots X_N$ , from  $k$  to  $j$ , to Chart;

c. ARC COMPLETION:

**for each** arc  $X \rightarrow X_1 \dots X_N C \bullet$  added in step (a) or step (b) **do**

Move completed constituent  $X$  to Agenda;

**end if**

**end loop**

# Problem with bottom-up chart parsing

- It ignores useful top-down knowledge (rule contexts).

```
>>> nltk.app.chartparser()
```

Add lexical ambiguity to defaults:

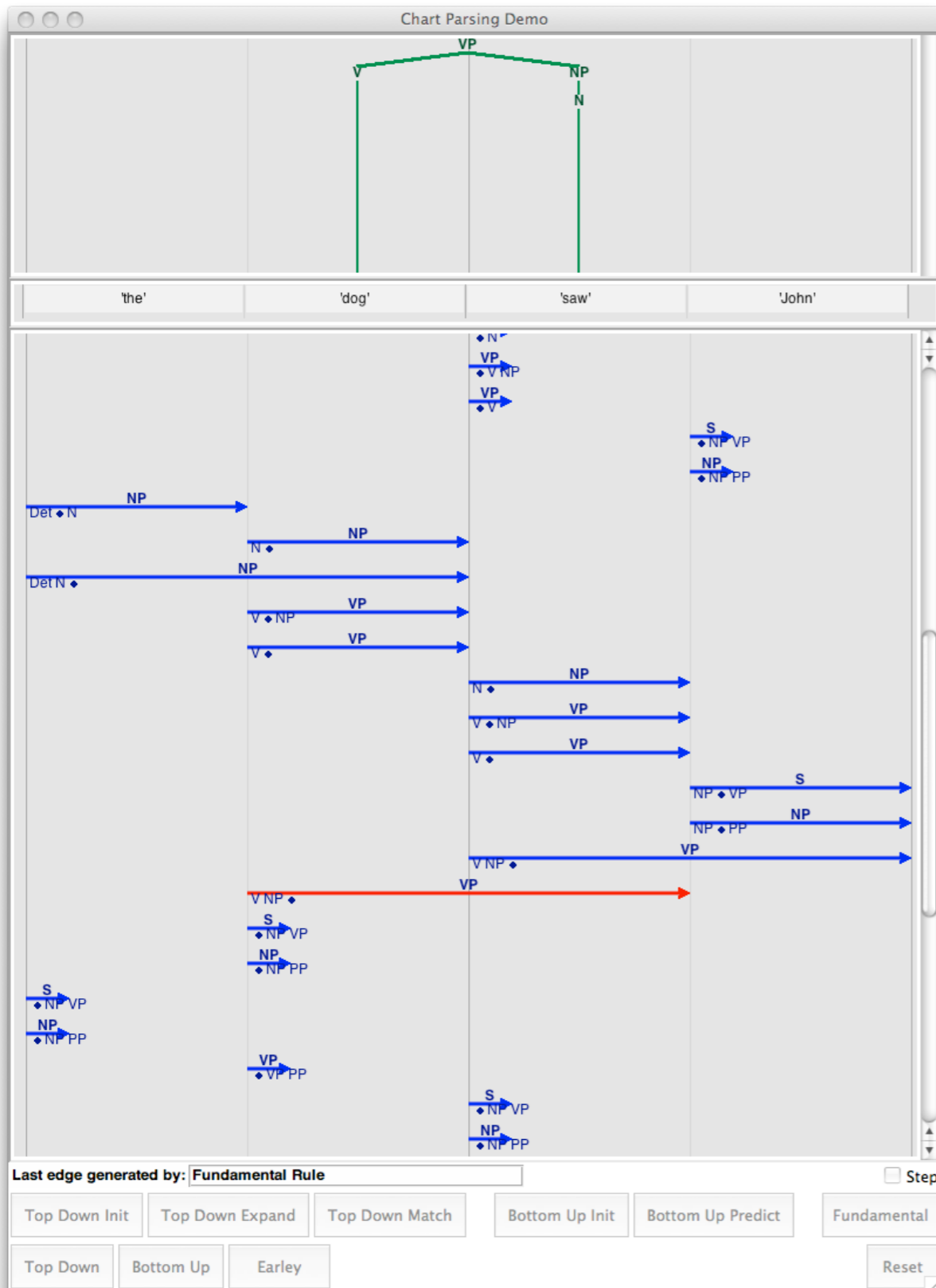
$N \rightarrow \textit{saw}$

$V \rightarrow \textit{dog}$

$NP \rightarrow N$

Parse bottom-up:

*the dog saw John*





# Top-down chart parsing

- Same as bottom-up, except **new** arcs are added to chart **only** if they are based on predictions from **existing** arcs.
- Initialize chart with unstarted active arcs for  $S$ .
  - $S \rightarrow \bullet X Y$
  - $S \rightarrow \bullet Z Q$
- Whenever an active arc is added, also add unstarted arcs for its next needed constituent.

```
>>> nltk.app.chartparser()
```

Add lexical ambiguity to defaults:

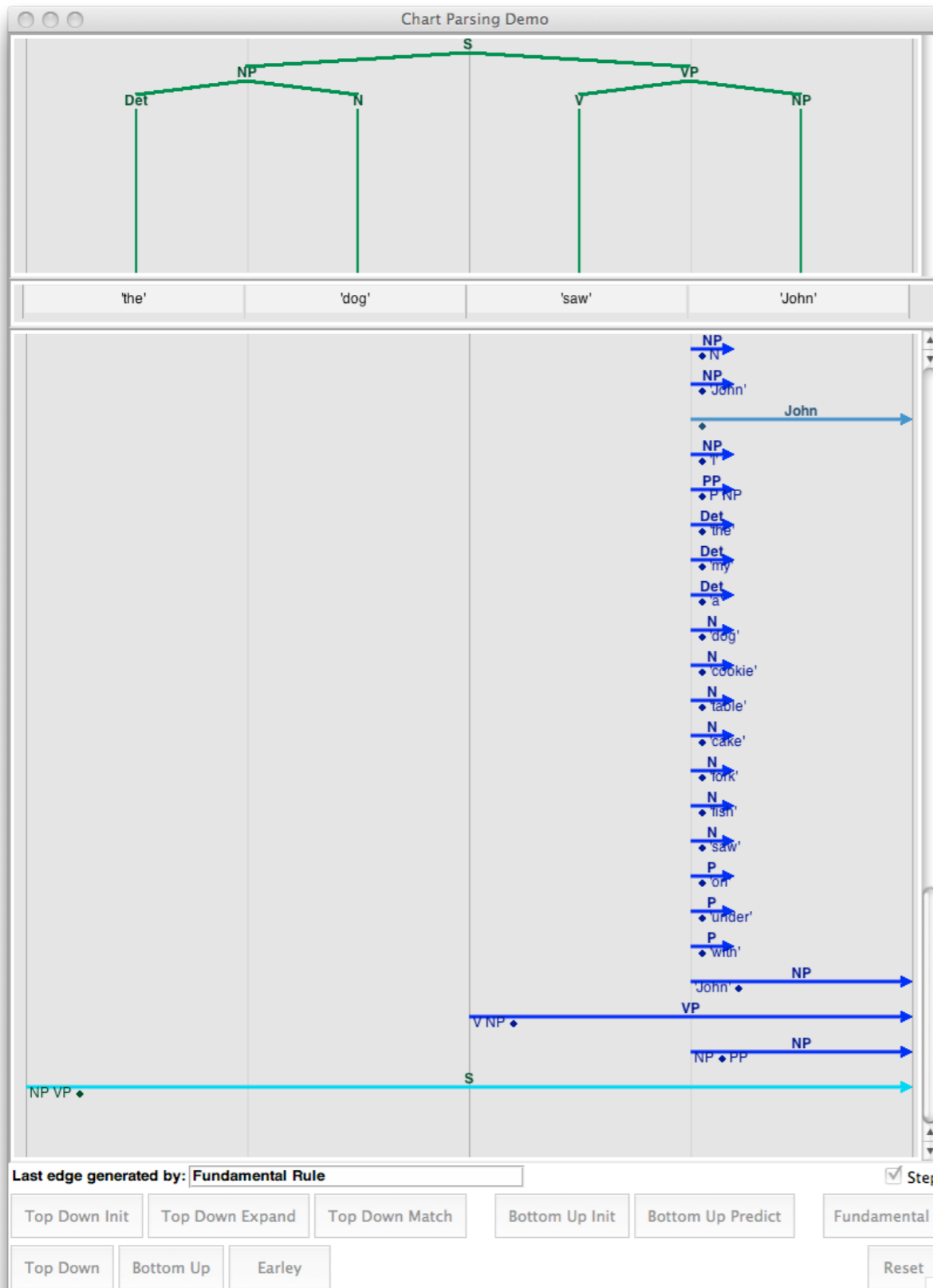
$N \rightarrow \textit{saw}$

$V \rightarrow \textit{dog}$

$NP \rightarrow N$

Parse top-down:

*the dog saw John*



# Top-down chart parsing

## Algorithm the first

INITIALIZE:

```
set Agenda = list of all possible categories of each input word  
                (in order of input);  
set n = length of input;  
set Chart = ();  
for each grammar rule  $S \rightarrow X_1 \dots X_N$  do  
    Add arc  $S \rightarrow \bullet X_1 \dots X_N$  to Chart at position 0;  
    apply TOP-DOWN ARC ADDITION [step (a') below] to the new arc;  
end for
```

ITERATE:

```
loop  
    if Agenda = () then  
        if there is at least one S constituent from 0 to n then  
            return SUCCESS  
        else  
            return FAIL  
        end if  
    else ...
```

# Top-down chart parsing

## Algorithm the second

**Set**  $C_{i,j} = \text{First}(\text{Agenda});$  */\* Remove first item from agenda. \*/*  
*/\*  $C_{i,j}$  is a completed constituent of type C from position i to position j \*/*  
**Add**  $C_{i,j}$  to Chart;

ARC UPDATE:

b. ARC EXTENSION (FUNDAMENTAL RULE):

**for each** arc  $X \rightarrow X_1 \dots \bullet C \dots X_N$ , from  $k$  to  $i$ , **do**

**Add** arc  $X \rightarrow X_1 \dots C \bullet \dots X_N$ , from  $k$  to  $j$ , to Chart;

a'. TOP-DOWN ARC ADDITION (PREDICTION):

*/\* Recursive: until no new arcs can be added \*/*

**for each** arc  $X \rightarrow X_1 \dots \bullet X_L \dots X_N$ , from  $k$  to  $j$ , added in step (b) or (a'), **do**

**Add** arc  $X_L \rightarrow \bullet Y_1 \dots Y_M$ , from  $j$  to  $j$ , to Chart;

c. ARC COMPLETION:

**for each** arc  $X \rightarrow X_1 \dots X_N C \bullet$  added in step (b) **do**

**Move** completed constituent  $X$  to Agenda;

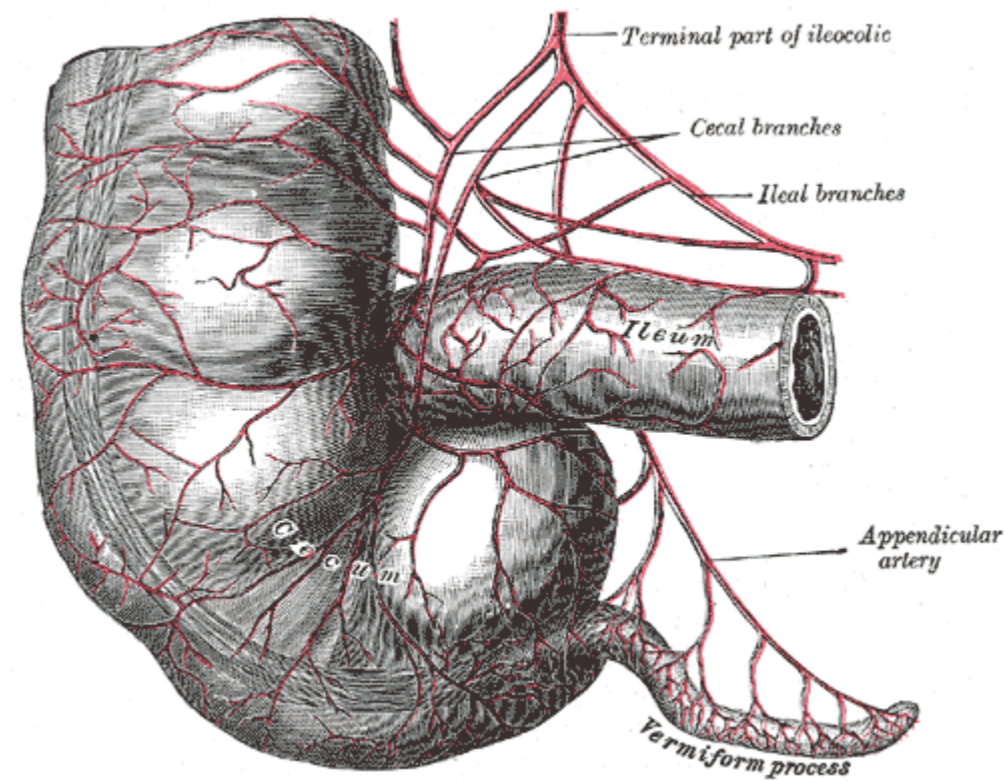
**end if**

**end loop**

# Notes on chart parsing

- Chart parsing separates:
  1. Policy for selecting constituent from agenda;
  2. Policy for adding new arcs to chart;
  3. Policy for initializing chart and agenda.
- “Top-down” and “bottom-up” now refer to arc-addition rule.
  - Initialization rule gives bottom-up aspect in either case.
- Polynomial algorithm ( $\theta(n^3)$ ), instead of exponential.

# Appendix



1 Paint 2 the 3 wall 4 with 5 cracks 6

Chart: Completed arcs

Grammar

$S \rightarrow V NP$   
 $S \rightarrow V NP PP$   
 $NP \rightarrow (Art) N$   
 $NP \rightarrow NP PP$   
 $PP \rightarrow P NP$

{V, Art, N, P, N}

Agenda

1 2 3 4 5 6

Chart: Active arcs

1 Paint 2 the 3 wall 4 with 5 cracks 6

V

1 2 3 4 5 6

$S \rightarrow V \bullet NP$   
 $S \rightarrow V \bullet NP PP$

Chart: Completed arcs

Grammar

$S \rightarrow V NP$   
 $S \rightarrow V NP PP$   
 $NP \rightarrow (Art) N$   
 $NP \rightarrow NP PP$   
 $PP \rightarrow P NP$

Chart: Active arcs

{V, Art, N, P, N}  
{Art, N, P, N}

Agenda



1 Paint 2 the 3 wall 4 with 5 cracks 6

|   |     |
|---|-----|
| V | Art |
|---|-----|



Chart: Completed arcs

Grammar

- $S \rightarrow V NP$
- $S \rightarrow V NP PP$
- $NP \rightarrow (Art) N$
- $NP \rightarrow NP PP$
- $PP \rightarrow P NP$

Chart: Active arcs

- {V, Art, N, P, N}
- {Art, N, P, N}
- {N, P, N}

Agenda

1 Paint 2 the 3 wall 4 with 5 cracks 6

|   |     |   |
|---|-----|---|
| V | Art | N |
|---|-----|---|

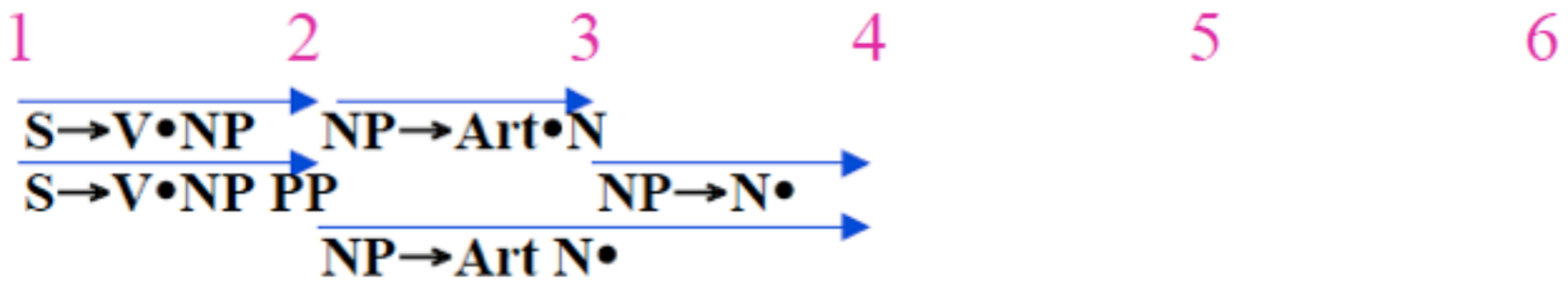


Chart: Completed arcs

- Grammar
- $S \rightarrow V NP$
  - $S \rightarrow V NP PP$
  - $NP \rightarrow (Art) N$
  - $NP \rightarrow NP PP$
  - $PP \rightarrow P NP$

Chart: Active arcs

- {V, Art, N, P, N}
- {Art, N, P, N}
- {N, P, N}
- {P, N}
- {NP, NP, P, N}

Agenda

1 Paint 2 the 3 wall 4 with 5 cracks 6

|   |                 |   |
|---|-----------------|---|
| V | Art             | N |
|   | NP <sub>1</sub> |   |

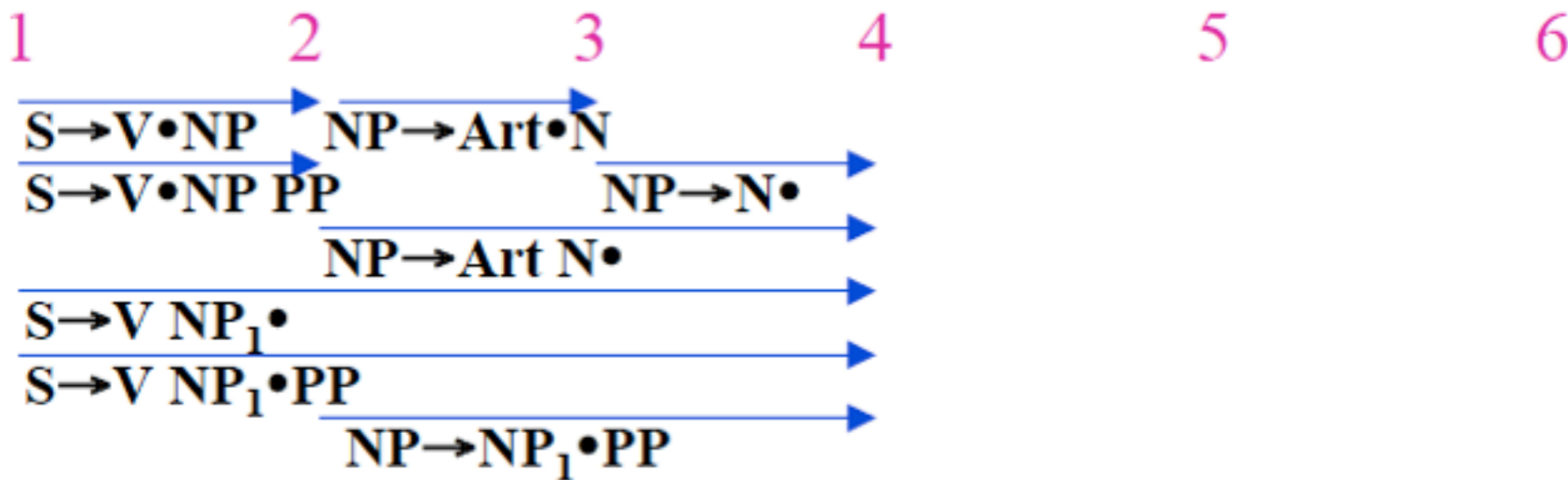


Chart: Completed arcs

Grammar

$S \rightarrow V NP$   
 $S \rightarrow V NP PP$   
 $NP \rightarrow (Art) N$   
 $NP \rightarrow NP PP$   
 $PP \rightarrow P NP$

Chart: Active arcs

$\{V, Art, N, P, N\}$   
 $\{Art, N, P, N\}$   
 $\{N, P, N\}$   
 $\{P, N\}$   
 $\{NP, NP, P, N\}$   
 $\{NP, P, N\}$   
 $\{S, NP, P, N\}$

Agenda

1 Paint 2 the 3 wall 4 with 5 cracks 6

|                |                 |                 |
|----------------|-----------------|-----------------|
| V              | Art             | N               |
|                | NP <sub>1</sub> |                 |
| S <sub>1</sub> |                 |                 |
|                |                 | NP <sub>2</sub> |

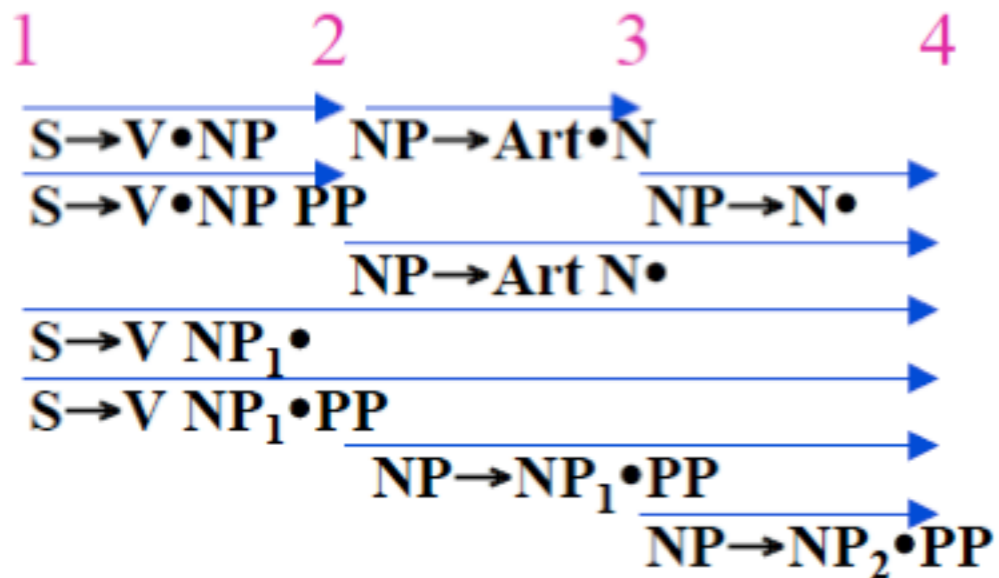


Chart: Completed arcs

Grammar

$S \rightarrow V NP$   
 $S \rightarrow V NP PP$   
 $NP \rightarrow (Art) N$   
 $NP \rightarrow NP PP$   
 $PP \rightarrow P NP$

Chart: Active arcs

$\{V, Art, N, P, N\}$   
 $\dots$   
 $\dots$   
 $\{S, NP, P, N\}$   
 $\{NP, P, N\}$   
 $\{P, N\}$

Agenda

1 Paint 2 the 3 wall 4 with 5 cracks 6

|                |                 |                 |   |                 |
|----------------|-----------------|-----------------|---|-----------------|
| V              | Art             | N               | P | N               |
|                | NP <sub>1</sub> |                 |   | NP <sub>3</sub> |
| S <sub>1</sub> |                 |                 |   |                 |
|                |                 | NP <sub>2</sub> |   |                 |

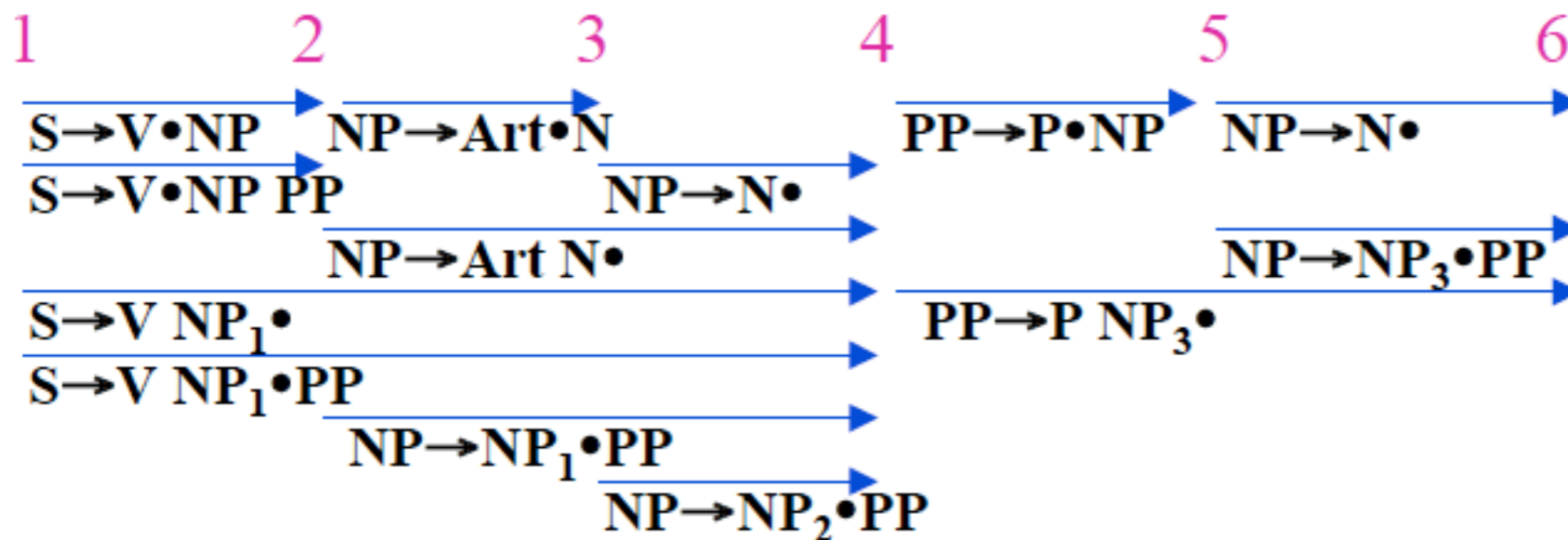


Chart: Completed arcs

Chart: Active arcs

Grammar

$S \rightarrow V NP$   
 $S \rightarrow V NP PP$   
 $NP \rightarrow (Art) N$   
 $NP \rightarrow NP PP$   
 $PP \rightarrow P NP$

{V, Art, N, P, N}  
 ...  
 ...  
 {P, N}  
 {N}  
 {}  
 {NP}  
 {}  
 {PP}

Agenda

1 Paint 2 the 3 wall 4 with 5 cracks 6

|                |                 |   |                 |                 |
|----------------|-----------------|---|-----------------|-----------------|
| V              | Art             | N | P               | N               |
|                | NP <sub>1</sub> |   |                 | NP <sub>3</sub> |
| S <sub>1</sub> |                 |   | PP <sub>1</sub> |                 |
|                | NP <sub>2</sub> |   |                 |                 |

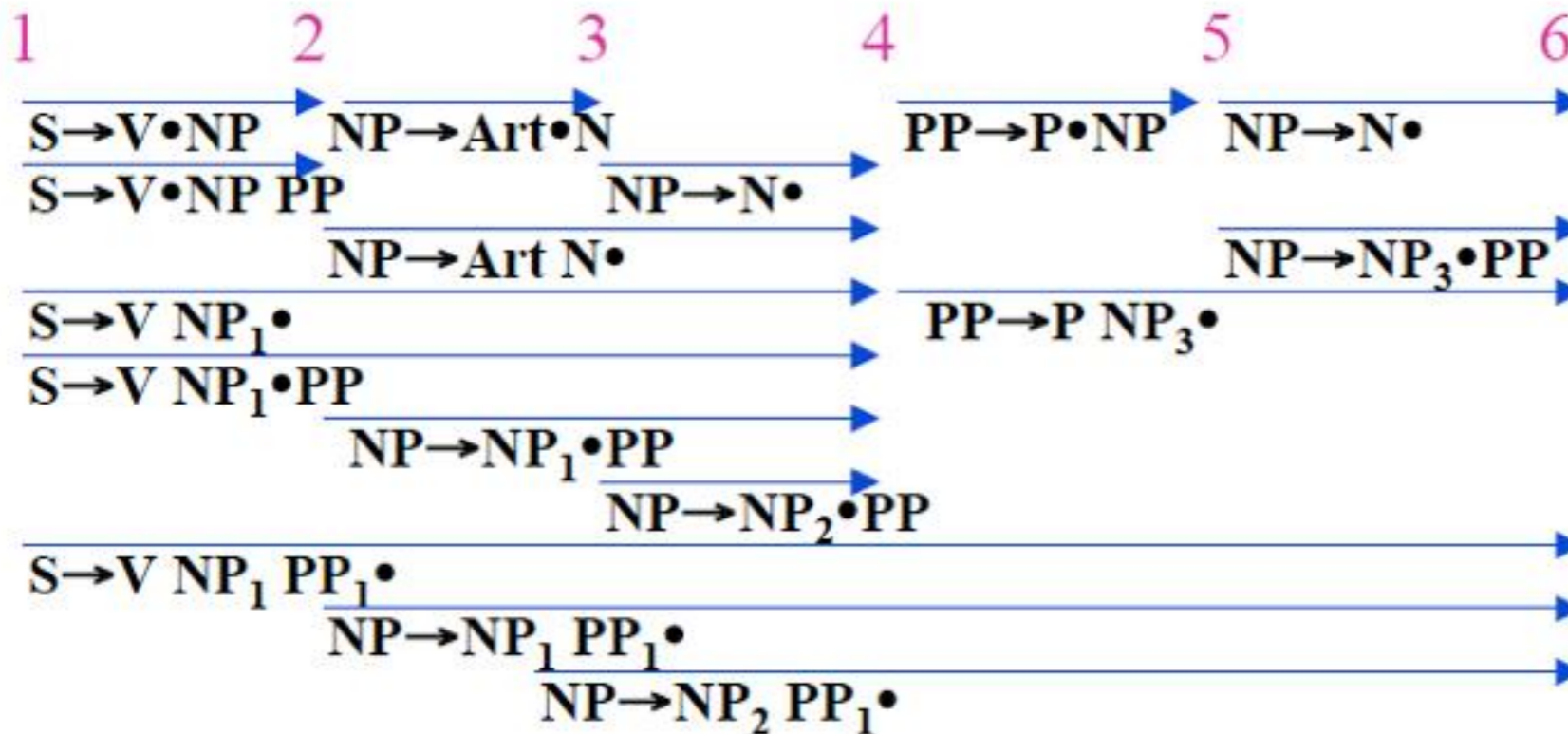


Chart: Completed arcs

Grammar

$S \rightarrow V NP$   
 $S \rightarrow V NP PP$   
 $NP \rightarrow (Art) N$   
 $NP \rightarrow NP PP$   
 $PP \rightarrow P NP$

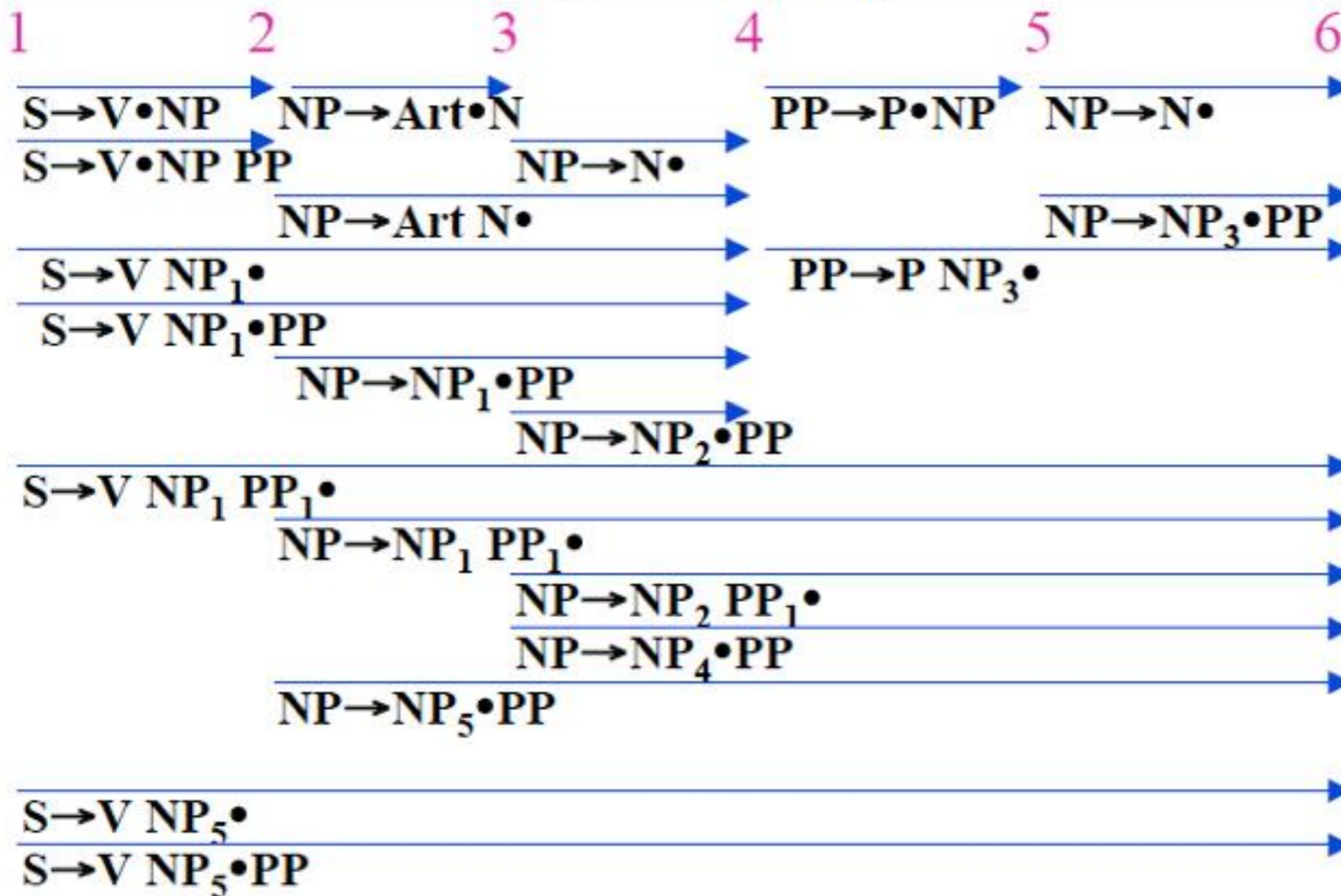
Chart: Active arcs

{V, Art, N, P, N}  
 ...  
 ...  
 {PP}  
 {}  
 {NP, NP, S}

Agenda

1 Paint 2 the 3 wall 4 with 5 cracks 6

|   |                 |   |                 |                 |
|---|-----------------|---|-----------------|-----------------|
| V   | Art             | N | P               | N               |
|   | NP <sub>1</sub> |   |                 | NP <sub>3</sub> |
| S <sub>1</sub>  |                 |   | PP <sub>1</sub> |                 |
|   | NP <sub>2</sub> |   | NP <sub>4</sub> |                 |
|   | NP <sub>5</sub> |   |                 |                 |
| S <sub>2</sub> (S → V NP <sub>5</sub> )                 |                 |   |                 |                 |
| S <sub>3</sub> (S → V NP <sub>1</sub> PP <sub>1</sub> ) |                 |   |                 |                 |



Grammar

$S \rightarrow V NP$   
 $S \rightarrow V NP PP$   
 $NP \rightarrow (Art) N$   
 $NP \rightarrow NP PP$   
 $PP \rightarrow P NP$

$\{V, Art, N, P, N\}$   
 $\dots$   
 $\dots$   
 $\{NP, NP, S\}$   
 $\{NP, S\}$   
 $\{S\}$   
 $\{S, S\}$   
 $\{S\}$   
 $\}$

Agenda

Chart: Completed arcs

Chart: Active arcs