CSC2415 Winter 2013 Homework Assignment 1

1. To within a constant factor, what is the worst case length of the chain between α and γ in the proof of Theorem 2.9?

2. Prove Lemma 2.11. Carefully justify the steps of your proof.

3. For any positive integer k, the k-cas operation takes as input k different objects $r_1, \ldots, r_k$, and $2k$ values $old_1, \ldots, old_k$, and $new_1, \ldots, new_k$.
   If the value of r_i is old_i for all $i \in \{1, \ldots, k\}$ then it sets the value of r_i to new_i for all $i \in \{1, \ldots, k\}$. Otherwise, the values of $r_1, \ldots, r_k$ remain unchanged.
   In either case, it returns the vector of values of $r_1, \ldots, r_k$ immediately beforehand.

   Prove that any synchronous, n-process algorithm for COLLECT that uses only k-cas and write requires $\Omega(\log_k n)$ rounds in the worst case.

4. Prove that any 3-process implementation of GetTS from registers that satisfies solo-termination requires at least 2 registers.

5. A *resettable consensus* object takes values from $\mathbb{N} \cup \{\bot\}$ and supports two operations:

   - *reset*, which sets the value of the object to $\bot$ and returns *ack*, and
   - *propose(v)*, which sets the value of the object to v, if the object has value $\bot$, and, otherwise, leaves the value of the object unchanged. In either case, it returns the (new) value of the object.

   Prove that any implementation of a counter shared by n processes using only resettable consensus objects requires at least $n-1$ resettable consensus objects and, in the worst case, a READ takes at least $n-1$ steps.