Heuristic Search ($A^*$)

Primary concerns in heuristic search:

- Completeness
- Optimality
- Time complexity
- Space complexity
- Practical computational effort

We guide the search for optimality and to reduce the computational load.
Let $h^*(n)$ be the cost of an optimal path from $n$ to a goal node.

- A heuristic is admissible if $h(n) \leq h^*(n)$ for all nodes $n$.
- Assumes $h(g) = 0$ for goal nodes $g$
- Assumes $c(n_1 \rightarrow n_2) > 0$ for any adjacent nodes $n_1$ and $n_2$
- Intuition: will not miss promising paths
- $f(n) = g(n) + h(n) \leq g(n) + h^*(n)$

Inadmissible heuristics can lead to suboptimal results with $A^*$
Consistency

Needed to ensure that the first time we visit a node we have an optimal path to it.

- Is a stronger condition than $h(n) \leq h^*(n)$.
- A monotone/consistent heuristic satisfies the triangle inequality (for all nodes $n_1, n_2$): $h(n_1) \leq c(n_1 \rightarrow n_2) + h(n_2)$
- Note that there might be more than one transition (action) between $n_1$ and $n_2$, the inequality must hold for all of them.
- Monotonicity implies admissibility.

In practice we emphasize admissible heuristics. Note $h(n) = 0$ for all $n$ is a consistent heuristic (but not a great one).
Relaxation Heuristics

Typical problems enforce some restrictions on how actions are taken. Removing these provides admissible heuristics. Consider the following restrictions on the 8 puzzle:

- Tiles must be moved into blank space.
- Tiles must be moved into an adjacent space.

Relaxing these conditions gives us the manhattan distance and misplaced tiles heuristics in the case of the 8 puzzle.
Relaxation Heuristics: Misplaced Tiles ($h_1$)

Assume that any tile can be moved to any position. (Relax both adjacency and blank space conditions).

- Simply need to move each tile to the final position
- $h(n)$ will therefore be the number of misplaced tiles
- Admissible as each misplaced tile must be moved
Assume that any tiles can overlap.
(Relax the requirement to move tiles into the blank space).

- Must move each tile to its final position independently
- \( h(n) \) will be the sum of the vertical and horizontal displacement for each tile
- Admissible as any solution to the original problem is a solution to the relaxed problem

Note that \( h_1(n) \leq h_2(n) \leq h^*(n) \). The manhattan distance heuristic dominates the misplaced tiles heuristics.
Pattern Database Heuristics

Can derive heuristics from solutions to subproblems

- Perform a search on the smaller subproblem, store the result
- The maximum cost over all stored subproblems is the heuristic value
- If the subproblems are disjoint we can add the costs

This method can be very effective when feasible. When well constructed this can outperform the manhattan distance heuristic for the 8 puzzle.
Problems with $A^*$ search

$A^*$ with an admissible heuristic provides optimality.

- Has the same space complexity as BFS!

Can we do better?
**IDA*: Iterative Deepening A* Search

Try to gain the memory benefits of iterative deepening search

- Cutoff by $f(n)$ value instead of depth
- Next iteration should use smallest higher valued $f(n)$ seen

Potentially problematic with many distinct $f(n)$ values
Other memory saving searches

$MA^*$, $SMA^*$ (memory bounded $A^*$, simplified memory bounded $A^*$)  
Try to reduce memory consumption

- Drop the worst leaf node in the open set (highest $f(n)$ value)
- Store additional data in the parent node of the dropped node so that it can be reconstructed

Time problems can occur if we need to frequently reconstruct dropped nodes.
Limitations enforced on search so far

All searches so far have relied on the following properties:

- Observability
- Known Environment
- Determinism
- Discrete States