Practice Problems for CSC 412/2506 Midterm

1. Let $p(k)$ be a one-dimensional discrete distribution that we wish to approximate, with support on nonnegative integers. One way to fit an approximating distribution $q(k)$ is to minimize the Kullback-Leibler divergence:

   $$KL(p||q) = \sum_{k=0}^{\infty} p(k) \log \frac{p(k)}{q(k)}$$

   Show that when $q(k)$ is a Poisson distribution, this KL divergence is minimized by setting λ to the mean of $p(k)$.

   $$q(k) = \frac{\lambda^k \exp(-\lambda)}{k!}$$

   Solution:

2. Recall that the definition of an exponential family model is:

   $$f(x|\eta) = h(x)g(\eta)\exp(\eta^\top T(x))$$

   where:

   $$\begin{aligned} \eta \quad & \text{are the parameters} \\ T(x) \quad & \text{are the sufficient statistics} \\ h(x) \quad & \text{is the base measure} \\ g(\eta) \quad & \text{is the normalizing constant} \end{aligned}$$

   Consider the univariate Gaussian, with mean μ and precision $\lambda = 1/\sigma^2$ is:

   $$p(D|\mu, \lambda = \prod_{i=1}^{N} \left(\frac{\lambda}{2\pi}\right)^{1/2} \exp(-\frac{\lambda}{2}(x_i - \mu)^2)$$

   What are η and $T(x)$ for this distribution when it is represented in exponential family form?

   Solution:

3. Consider the DAG in Figure 1. List all variables that are independent of A given evidence on B. You can use the "Bayes ball" algorithm, the d-separation criterion, or the method of converting to an undirected graph (all should give the same result).

   Solution:

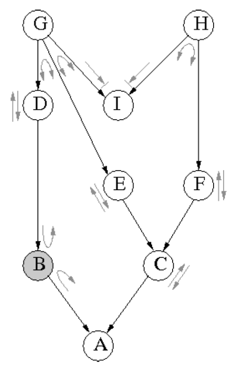

Figure 1: A directed graphical model.

4. Variable Elimination: Murphy 20.1

   Solution:

5. Consider a tree-structured factor graph over discrete variables, and suppose we wish to evaluate the joint distribution $[(x_a, x_b)$ associated with two variables x_a and x_b that do not belong to a common factor. Define a procedure for using the sum-product algirhtm to evaluate this joint distribution in which one of the variables is successively clamped to each of its allowed values.

   Solution: