CSC373 Summer '22
Tutorial 6
July 21, 2022

Q1 P vs NP vs co-NP

Are the following decision problems in P, NP, or co-NP? Give the strongest possible answer (i.e., if you can show that the decision problem is in P, use that instead of NP or co-NP).

1. **TRIANGLE**
Input: An undirected graph $G = (V, E)$.
Question: Does G contain a "triangle" (i.e., a subset of three vertices such that there is an edge between any two of them)?

2. **CLIQUE**
Input: An undirected graph $G = (V, E)$ and a positive integer k.
Question: Does G contain a k-clique (i.e., a subset of k vertices such that there is an edge between any two of them)?

3. **NON-ZERO**
Input: A set of integers S.
Question: Does every non-empty subset of S have non-zero sum?

4. **HAMILTONIAN-PATH (HP)**
Input: An undirected graph $G = (V, E)$.
Question: Does G contain a simple path that includes every vertex?

Q2 NP-Completeness I

Consider the Hamiltonian Cycle (HC) problem, which is similar to the HP problem described above.

HAMILTONIAN-CYCLE (HC)
Input: An undirected graph $G = (V, E)$.
Question: Does G contain a simple cycle that includes every vertex?

(a) The textbook CLRS shows that HC is is NP-complete (Subsection 34.5.3). Give a reduction from HC to HP (i.e., HC $\leq_p$ HP) to prove HP is also NP-complete.

(b) Suppose instead that we knew HP is NP-complete and wanted to use it to show that HC is NP-complete. Give a reduction from HP to HC (i.e., HP $\leq_p$ HC).

Q3 NP-Completeness II

Consider the following problem. A multiset allows repeated elements.

PARTITION

Input: A multiset S containing positive integers.

Question: Is there a partition of S into two multisets (i.e. $S_1, S_2 \subseteq S$ such that $S_1 \cap S_2 = \emptyset$ and $S_1 \cup S_2 = S$) whose elements have equal sum?

(a) Prove that PARTITION is in NP.

(b) Prove that PARTITION is NP-hard through a reduction from SUBSET-SUM.

SUBSET-SUM

Input: A multiset S containing positive integers and an integer W.

Question: Is there a subset $S' \subseteq S$ whose elements sum to W?