

# MOPO: Model-based Offline Policy Optimization

Tianhe Yu, Garrett Thomas, Lantao Yu, Stefano Ermon,  
James Zou<sup>1</sup>, Sergey Levine, Chelsea Finn, Tengyu Ma

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Claas A Voelcker

STA 4273 - University of Toronto

# Summary

- The title says it all
  - Model-based: learning environment models
  - Offline: learning with offline (precollected) data
  - Policy Optimization: learning a policy
- Review the background
- Review the theory
- Review the algorithm
- Poke at the weak spots

# Introduction

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## The challenge: Offline data

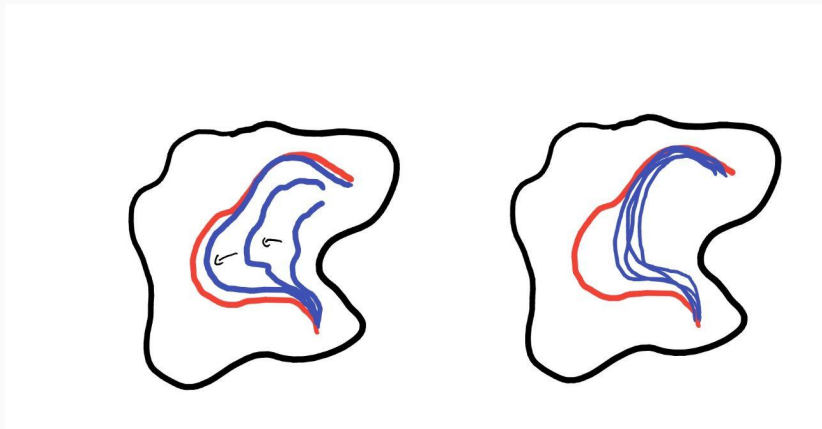


Figure 1: A crude visualization of offline RL (left online, right offline)

# The challenge: Offline data

## Challenges:

- Might not contain correct solution
- Intermediate policies could lead outside of data covered region
- Generalization of RL algorithms unclear

## Solutions:

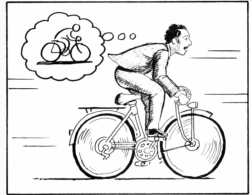
- Inverse reinforcement learning
- Regularization towards data distribution
- Hope for generalization
- Model-based RL?

# An answer? Model learning

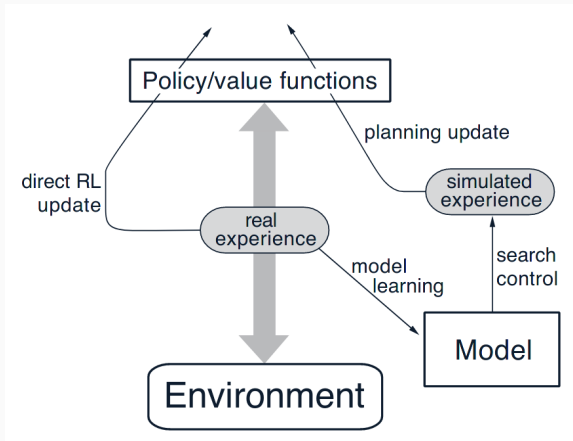
Why model learning?

- Supervised: more hopes of generalization
- Model can cover region of low data
- We can estimate model uncertainty

Classic algorithm: Dyna



**Figure 2:** Comic from Ha, Schmidhuber: "World Models", (<https://arxiv.org/pdf/1803.10122.pdf>)



**Figure 3:** Diagram from Sutton, Barto: "Reinforcement Learning: An Introduction", p.163, MIT Press 2018

# MBPO as offline-learning

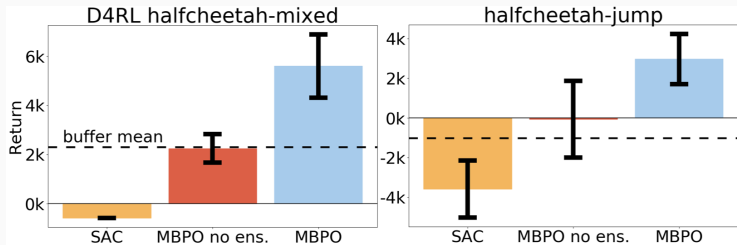


Figure 4: Comparison of previous methods on offline benchmarks, diagram from paper



# Offline optimized model-based RL

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# When to trust your model (revisited)

Disclaimer: Compressed notation for intuition, not rigorous

Try to quantify the error when executing policies  $\pi$  from one model  $\hat{T}$  in another  $T$

Expected discounted return :

$$\eta_T(\pi) := \mathbb{E}_T \left[ \sum \gamma^t r(s_t, a_t) \right]$$

Difference in value function :

$$G^\pi(s, a) := \mathbb{E}_{s' \sim \hat{T}(s, a)} [V_T^\pi(s')] - \mathbb{E}_{s' \sim T(s, a)} [V_T^\pi(s')]$$

# How good are we sure to be?

Estimate expected return under true dynamics  $T$

$$\begin{aligned}\eta_{\hat{T}}(\pi) - \eta_T(\pi) &= \gamma \mathbb{E}_{\hat{T}}^{\pi} \left[ \sum \gamma^t G_{\hat{T}}^{\pi}(s_t, a_t) \right] \\ \eta_T(\pi) &= \mathbb{E}_{\hat{T}}^{\pi} \left[ \sum \gamma^t (r(s_t, a_t) - \gamma G_{\hat{T}}^{\pi}(s_t, a_t)) \right] \\ &\geq \mathbb{E}_{\hat{T}}^{\pi} \left[ \sum \gamma^t (r(s_t, a_t) - \gamma |G_{\hat{T}}^{\pi}(s_t, a_t)|) \right]\end{aligned}$$

Need  $|G^{\pi}(s, a)| = |\mathbb{E}_{s' \sim \hat{T}(s, a)}[V^{\pi}(s')] - \mathbb{E}_{s' \sim T(s, a)}[V^{\pi}(s')]|$

$$|G_{\hat{T}}^{\pi}(s, a)| \leq \sup_{V \in \mathcal{F}} |\mathbb{E}_{s' \sim T}[V(s'|s, a)] - \mathbb{E}_{s' \sim \hat{T}}[V(s'|s, a)]| = d_{\mathcal{F}}(\hat{T}(s, a), T(s, a))$$

- For  $\mathcal{F}$  bounded: Total variation distance
- For  $\mathcal{F}$  Lipschitz-smooth: Wasserstein distance

## How good are we sure to be?

Idea: expected return in  $T$  is lower bounded by:

$$\mathbb{E}_{\hat{T}} \left[ \sum \gamma^t \left( r(s_t, a_t) - \gamma d_{\mathcal{F}}(\hat{T}(s_t, a_t), T(s_t, a_t)) \right) \right] \quad (1)$$

- new MDP with  $\tilde{r}(s, a) = r(s, a) - \gamma d_{\mathcal{F}}(\hat{T}(s, a), T(s, a))$
- optimize policy here
- by previous, return will underestimate true return (achieve conservative learning)

Big problem: don't know  $T$  and therefore also not  $d_{\mathcal{F}}(T(\hat{s}, a), T(s, a))$

Idea: Find function  $u(s, a) \geq d_{\mathcal{F}}(\hat{T}(s, a), T(s, a))$  and define  $\tilde{r}(s, a) = r(s, a) - u(s, a)$

Making it work in practice

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# Uncertainty in ensemble NN

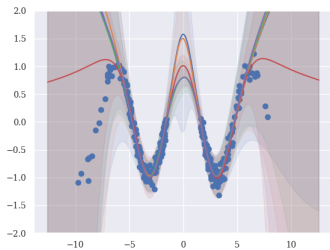
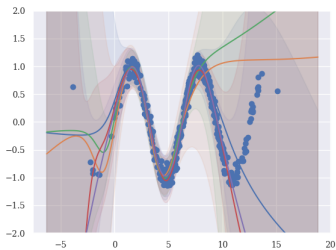
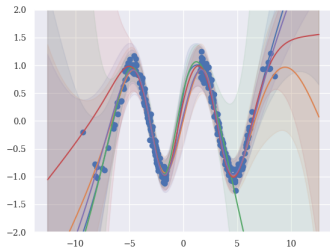
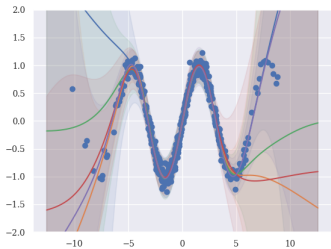
How do we get  $u$ ?

Core idea (reuses model from PETS, MBPO):

- Take  $n$  identical neural networks
- Encode  $p(y|x) = \mathcal{N}(\mu^i(s), \Sigma^i(s))$
- Train independently to minimize  $-\frac{1}{n} \sum \log p(y|x)|_{x, y \sim D}$
- Each network captures intrinsic randomness (aleatoric)
- Whole ensemble captures data uncertainty (epistemic)

Measure uncertainty with these

# Uncertainty in ensemble NN – Visualization



Code available at <https://colab.research.google.com/drive/>

# Implementation of a practical algorithm

**Require:**  $\lambda$ , rollout horizon  $h$ , rollout batch size  $b$ .

- 1: Train on batch data  $\mathcal{D}_{\text{env}}$  an ensemble of  $N$  probabilistic dynamics  $\{\hat{T}^i(s', r|s, a) = \mathcal{N}(\mu^i(s, a), \Sigma^i(s, a))\}_{i=1}^N$ .
- 2: Initialize policy  $\pi$  and empty replay buffer  $\mathcal{D}_{\text{model}} \leftarrow \emptyset$ .
- 3: **for** epoch  $1, 2, \dots$  **do**
- 4:     **for**  $1, 2, \dots, b$  (in parallel) **do**
- 5:         Sample state  $s_1$  from  $\mathcal{D}_{\text{env}}$  for init
- 6:         **for**  $j = 1, 2, \dots, h$  **do**
- 7:             Sample an action  $a_j \sim \pi(s_j)$ .
- 8:             Pick  $\hat{T}$  from  $\{\hat{T}^i\}_{i=1}^N$  and sample  $s_{j+1}, r_j \sim \hat{T}(s_j, a_j)$ .
- 9:             Compute  $\tilde{r}_j = r_j - \lambda \max_{i=1}^N \|\Sigma^i(s_j, a_j)\|_F$ .
- 10:             Add sample  $(s_j, a_j, \tilde{r}_j, s_{j+1})$  to  $\mathcal{D}_{\text{model}}$
- 11:     Drawing samples from  $\mathcal{D}_{\text{env}} \cup \mathcal{D}_{\text{model}}$ , update  $\pi$ .



## Does this work with the theory

We have:

$N$  probabilistic dynamics  $\{\hat{T}^i(s', r|s, a) = \mathcal{N}(\mu^i(s, a), \Sigma^i(s, a))\}_{i=1}^N$

We estimate  $\tilde{r}$  as

$$\tilde{r}_j = r_j - \lambda \max_i \|\Sigma^i(s_j, a_j)\|_F = r(s, a) - \gamma u(s, a)$$

Reminder:

$$|G_{\tilde{r}}^{\pi}(s, a)| \leq d_{\mathcal{F}}(\hat{T}(s, a), T(s, a)) \stackrel{?}{=} \lambda \max_i \|\Sigma^i(s_j, a_j)\|_F$$

Uncertainty estimate proposed in paper and tested:

$$u(s, a) = \lambda \max_{i=1} \|\Sigma^i(s_j, a_j)\|_F$$

$$u(s, a) = \lambda \max_{i,j} \|\mu_i - \mu_j\|_2$$

In experiments, max variance performed better than disagreement...

What about (alternative proposal):

$$u(s, a) = \lambda \text{Var}(\text{ensemble})(s, a) = \\ \lambda \left( \sum \sigma_i^2(s, a) + \sum \mu_i^2(s, a) - \left( \sum \mu_i(s, a) \right)^2 \right)$$

Open question: Relationship of uncertainty and divergence measure

What do we take away?

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# Summary

- Interesting theory, solid foundation
  - Model-based RL can shine in offline settings
  - Clear connection between model error and expected return
- Empirically very strong algorithm
  - Works very well when requiring OOD data for optimal policy
  - Results mostly skipped here because there were no nice graphs
- Very little connection between theory and empirical work (also noted by reviewers)
- Uncertainty measurement drives even larger gaps between theory and empirical algorithm