CSC 411: Lecture 19: Reinforcement Learning

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Today

- Learn to play games
- Reinforcement Learning

Playing Games: Atari

https://www.youtube.com/watch?v=V1eYniJ0Rnk

Playing Games: Super Mario

https://www.youtube.com/watch?v=wfL4L_l4U9A

Making Pancakes!

https://www.youtube.com/watch?v=W_gxLKSsSIE

Reinforcement Learning Resources

- RL tutorial – on course website
- *Reinforcement Learning: An Introduction*, Sutton & Barto Book (1998)

Reinforcement Learning

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  - Supervised: correct outputs

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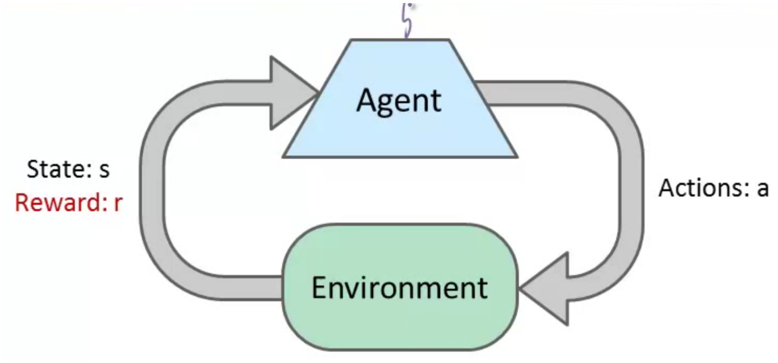
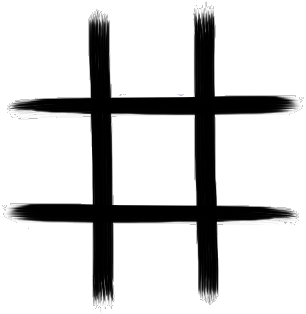
- More realistic learning scenario:
  - Continuous stream of input information, and actions
  - Effects of action depend on state of the world
  - Obtain reward that depends on world state and actions
    - not correct response, just some feedback

Reinforcement Learning



[pic from: Peter Abbeel]

Example: Tic Tac Toe, Notation



environment

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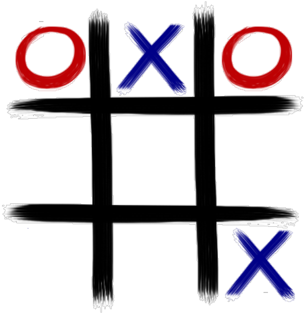

(current)
state

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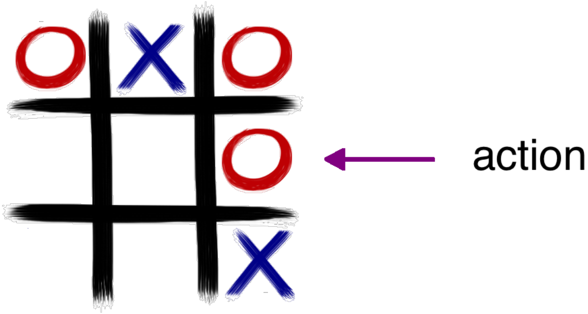

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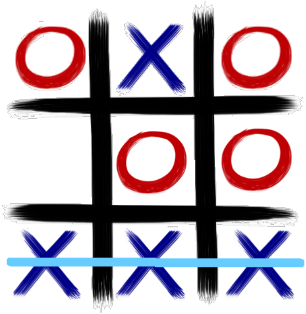

reward

(here: -1)

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  - Policy π: agent's behaviour function
  - Value function: how good is each state and/or action
  - Model: agent's representation of the environment

Policy

- A policy is the agent's behaviour.
- It's a selection of which action to take, based on the current state
- Deterministic policy: $a = \pi(s)$
- Stochastic policy: $\pi(a|s) = P[a_t = a | s_t = s]$

[Slide credit: D. Silver]

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- γ is called a discount rate, and it is always $0 \leq \gamma \leq 1$
- If γ close to 1, rewards further in the future count more, and we say that the agent is "farsighted"
- γ is less than 1 because there is usually a time limit to the sequence of actions needed to solve a task (we prefer rewards sooner rather than later)

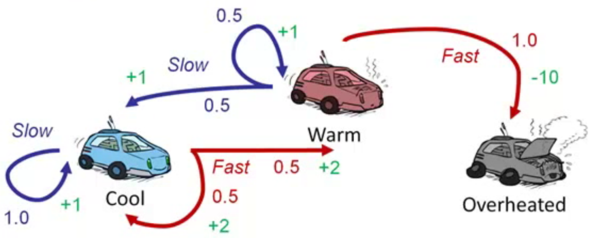
[Slide credit: D. Silver]

Model

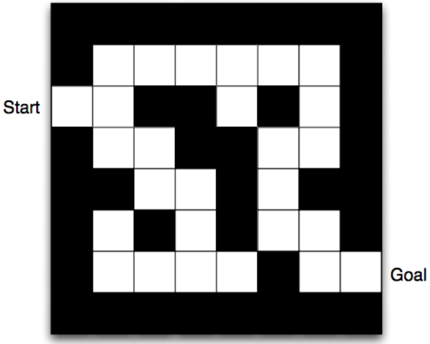
- The model describes the environment by a distribution over rewards and state transitions:

$$P(s_{t+1} = s', r_{t+1} = r' | s_t = s, a_t = a)$$

- We assume the Markov property: the future depends on the past only through the current state

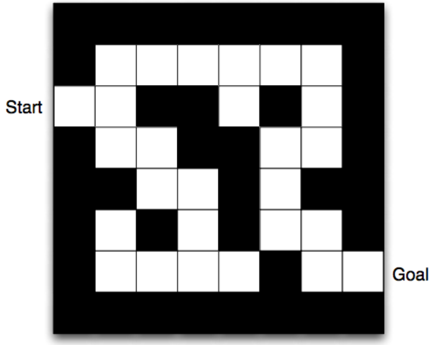


Maze Example



- Rewards:

Maze Example



- Rewards: -1 per time-step
- Actions:

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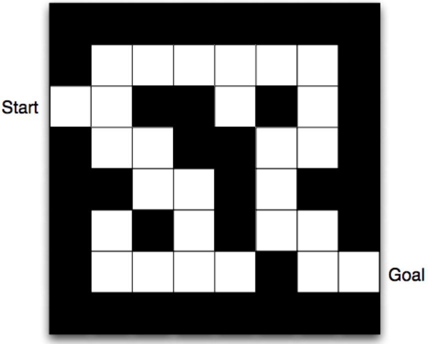

- Rewards: -1 per time-step
- Actions: N, E, S, W
- States:

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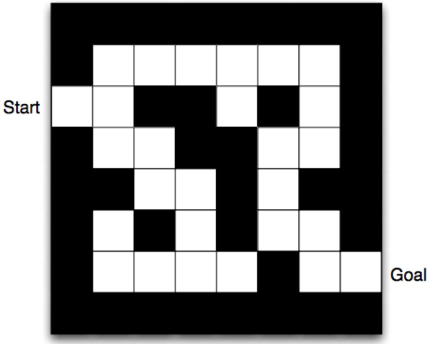

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[Slide credit: D. Silver]

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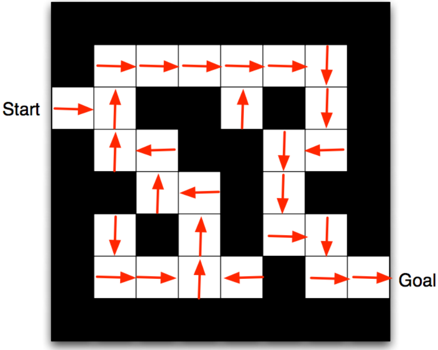

- Arrows represent policy $\pi(s)$ for each state s

[Slide credit: D. Silver]

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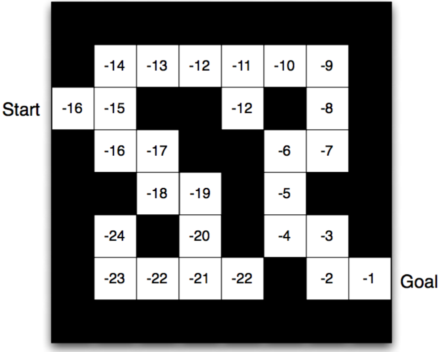

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- In tic-tac-toe, since state space is tractable, can use a table to represent value function

RL & Tic-Tac-Toe

- Each board position (taking into account symmetry) has some probability

State	Probability of a win (Computer plays "o")
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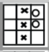
	1.0
	0.0
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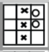	0.0
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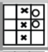	0.0
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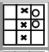	0.0
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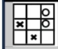	0.5
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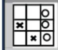	1.0
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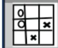	0.5
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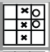	0.0
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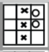	0.0
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  - After many games value function will represent true probability of winning from each state
- Can try alternative policy: sometimes select moves randomly (exploration)

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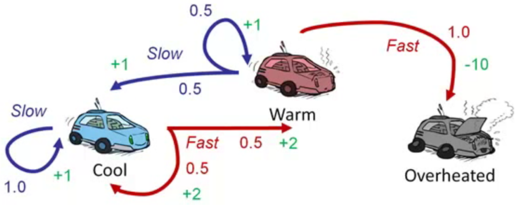

[Pic: P. Abbeel]

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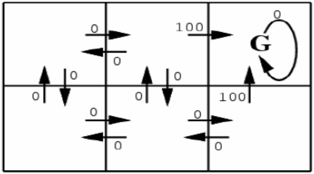
- Standard MDP problems:
  1. Planning: given complete Markov decision problem as input, compute policy with optimal expected return
  2. Learning: We don't know which states are good or what the actions do. We must try out the actions and states to learn what to do

[P. Abbeel]

Example of Standard MDP Problem



$r(s, a)$ (immediate reward)

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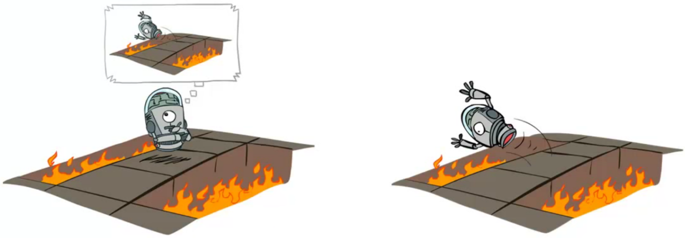

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We will focus on learning, but discuss planning along the way

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- Without losing too much reward along the way
- Since we do not have complete knowledge of the world, taking what appears to be the optimal action may prevent us from finding better states/actions
- Interesting trade-off:
  - immediate reward (exploitation) vs. gaining knowledge that might enable higher future reward (exploration)

Examples

- Restaurant Selection
  - Exploitation: Go to your favourite restaurant
  - Exploration: Try a new restaurant
- Online Banner Advertisements
  - Exploitation: Show the most successful advert
  - Exploration: Show a different advert
- Oil Drilling
  - Exploitation: Drill at the best known location
  - Exploration: Drill at a new location
- Game Playing
  - Exploitation: Play the move you believe is best
  - Exploration: Play an experimental move

[Slide credit: D. Silver]

MDP Formulation

- Goal: find policy π that maximizes expected accumulated future rewards $V^{\pi}(s_t)$, obtained by following π from state s_t:

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$$

- Game show example:
  - assume series of questions, increasingly difficult, but increasing payoff
  - choice: accept accumulated earnings and quit; or continue and risk losing everything
- Notice that:

$$V^{\pi}(s_t) = r_t + \gamma V^{\pi}(s_{t+1})$$

What to Learn

- We might try to learn the function V (which we write as V^*)

$$V^*(s) = \max_a [r(s,a) + \gamma V^*(\delta(s,a))]$$

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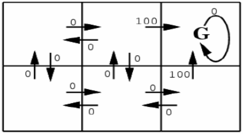
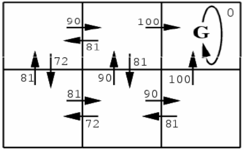
- Q is then the evaluation function we will learn

$\gamma = 0.9$



$r(s,a)$ (immediate reward) values



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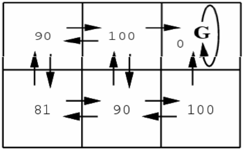
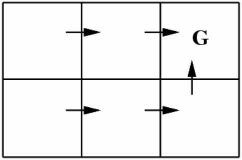


$V^*(s)$ values

$$V^*(s_5) = 0 + \gamma 100 + \gamma^2 0 + \ldots = 90$$



One optimal policy

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$$\hat{Q}(s, a) \leftarrow r(s, a) + \gamma \max_{a'} \hat{Q}(s', a')$$

where s' is state resulting from applying action a in state s

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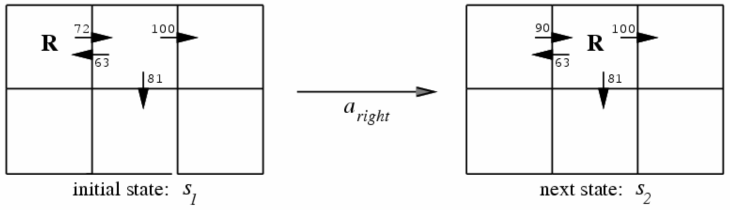
- If we get to absorbing state, restart to initial state, and run thru "Do forever" loop until reach absorbing state

Updating Estimated Q

- Assume the robot is in state s_1; some of its current estimates of Q are as shown; executes rightward move



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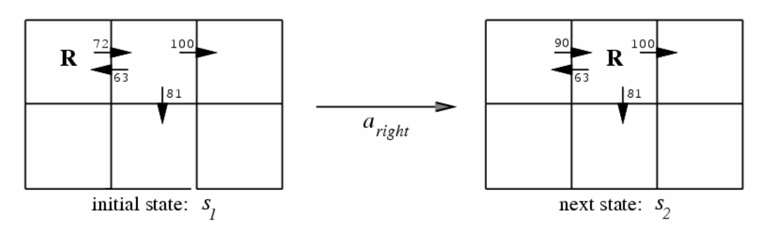

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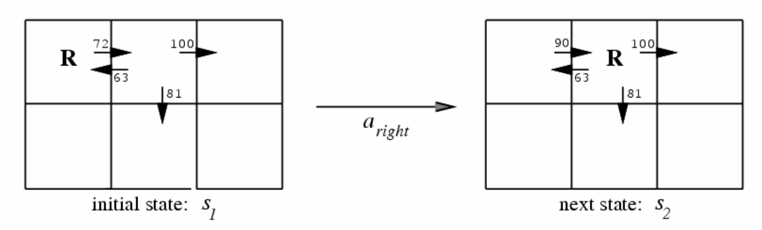

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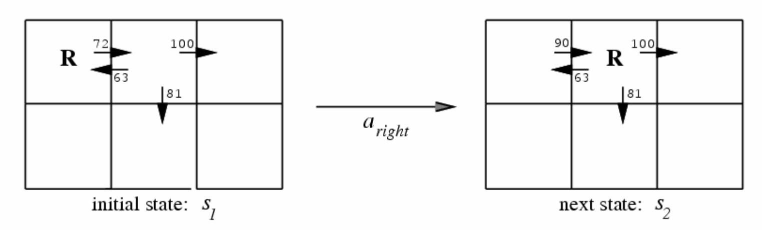

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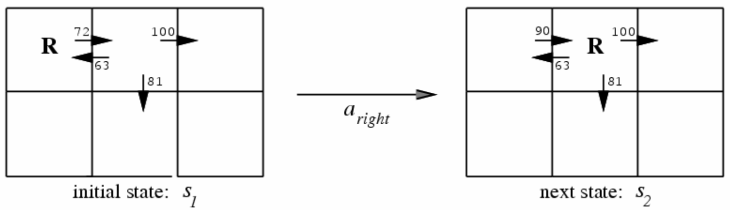

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- Notice that if rewards are non-negative, then $\hat{Q}$ values only increase from 0, approach true Q

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  4. Eventually propagate information from transitions with non-zero reward throughout state-action space

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  - more exploration early on, shift towards exploitation

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- So modify training rule to change more slowly

$$\hat{Q}(s,a) \leftarrow (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_n[r + \gamma \max_{a'} \hat{Q}_{n-1}(s',a')]$$

where s' is the state land in after s, and a' indexes the actions that can be taken in state s'

$$\alpha_n = \frac{1}{1 + \text{visits}_n(s,a)}$$

where visits is the number of times action a is taken in state s