Function Symbols in Prolog:

From Deductive Databases to Logic Programs

In logic, there are two kinds of objects: predicates and functions.

- Predicates represent statements about the world:

  `John hates Mary:  hates(john,mary).`

  `John is short:  short(john)`

(`hates` is a predicate symbol, `short(john)` is an atomic formula)

- Function terms represent objects in the world

  `the mother of Mary:  mother-of(mary)`

  `a rectangle of length 3 and width 4:`

  `rectangle(3,4)`

(`mother-of(mary)` is a function term, `rectangle` is a function symbol)

Function terms do not have values. In Prolog, they act as data structures:

let `p2(X,Y)` denote a point in 2-dim space
let `p3(X,Y,Z)` denote a point in 3-dim space.

Write a Prolog program, `SQDIST(Point1,Point2,D)`, that returns the square of the distance between two points. The program should work for 2- and 3-dim points.

Want:

```
SQDIST(p2(1,2), p2(3,5), D)
  returns D = (3-1)**2 + (5-2)**2
            = 4+9 = 13
and
SQDIST(p3(1,1,0), p3(2,2,3), D)
  returns D = (1-2)**2 + (1-2)**2 + (0-3)**2
            = 1+1+9 = 11
and
SQDIST(p2(0,0), p3(1,1,1), D)
  is undefined
```

Prolog Program:

```
(1) SQDIST(p2(X1,Y1), p2(X2,Y2), D)
      :- XD is X1-X2,
         YD is Y1-Y2,
         D is XD*XD + YD*YD.

(2) SQDIST(p3(X1,Y1,Z1), p3(X2,Y2,Z2), D)
       :- XD is X1-X2,
          YD is Y1-Y2,
          ZD is Z1-Z2,
          D is XD*XD + YD*YD + ZD*ZD.
```

Query: `SQDIST(p2(1,2), p2(3,5), D)`

This query unifies with the head of rule (1) with {X1\1, Y1\2, X2\3, Y2\5}

so, XD is X1-X2 = 1-3 = -2

YD is Y1-Y2 = 2-5 = -3

D is $(-2)^2 + (-3)^2 = 13$

So, D=13 is returned

Note: the query does not unify with the head of rule (2), so only rule (1) is used.

Prolog Program:

```
(1) SQDIST(p2(X1,Y1), p2(X2,Y2), D)
      :- XD is X1-X2,
         YD is Y1-Y2,
         D is XD*XD + YD*YD.

(2) SQDIST(p3(X1,Y1,Z1), p3(X2,Y2,Z2), D)
       :- XD is X1-X2,
          YD is Y1-Y2,
          ZD is Z1-Z2,
          D is XD*XD + YD*YD + ZD*ZD.
```

Query: `SQDIST(p3(1,1,0),p3(2,2,3),D).`

This query unifies with the head of rule (2), with {X1\1, Y1\1, Z1\0, X2\2, Y2\2, Z2\3}

so, XD is 1-2 = -1
YD is 1-2 = -1
ZD is 0-3 = -3
D is 1+1+9 = 11
So, D=11 is returned

Note: the query does not unify with the head of rule (1), so only rule (2) is used.

Prolog Program:

```
(1) SQDIST(p2(X1,Y1), p2(X2,Y2), D)
      :- XD is X1-X2,
         YD is Y1-Y2,
         D is XD*XD + YD*YD.

(2) SQDIST(p3(X1,Y1,Z1), p3(X2,Y2,Z2), D)
       :- XD is X1-X2,
          YD is Y1-Y2,
          ZD is Z1-Z2,
          D is XD*XD + YD*YD + ZD*ZD.
```

Query: `SQDIST(p2(0,0), p3(1,1,1), D).`

Note: this query does not unify with any rule, so Prolog simply returns `no`, i.e., no answers for `D`.

Returning Function Terms as Answers

e.g., given a point, `p2(X,Y)`, return a new point with double the coordinates. *e.g.*,

Query: `double(p2(3,4),P)`
Answer:`P = p2(6,8).`

Prolog Program:

```
double(p2(X1,Y1), p2(X2,Y2))
   :- X2 is 2*X1,
      Y2 is 2*Y1.
```

In Plain English: if `X2 = 2*X1` and `Y2 = 2*Y1`, then the double of `p2(X1,Y1)` is `p2(X2,Y2)`.

An equivalent program using "=":

```
double(p2(X1,Y1), P)
   :- X2 is 2*X1, Y2 is 2*Y1,
      P = p2(X2,Y2).
```

Here, "=" is being used to assign a value to variable `P`. Try to avoid this!!!!! It reflects procedural thinking.

Sample Execution

Prolog Program:

```
double(p2(X1,Y1), p2(X2,Y2))
   :- X2 is 2*X1,
      Y2 is 2*Y1.
```

Query: `double(p2(3,4),P)`

The query unifies with the head of the rule, where the mgu is

{X1\3, Y1\4, P\p2(X2,Y2)}

The body of the rule then evaluates:

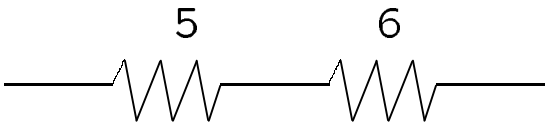
```
X2 is 2*X1,    i.e., 6
Y2 is 2*Y1,    i.e., 8
```

The mgu becomes {X1\3, Y1\4, P\p2(6,8)}.

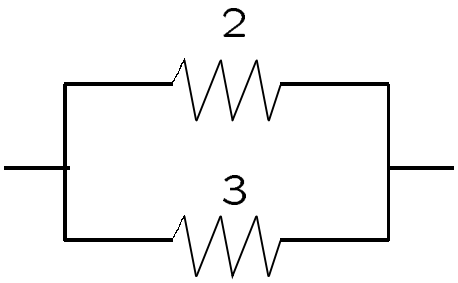
So, the answer is `P = p2(6,8).`

Recursion with Function Symbols

Example: Electrical circuits

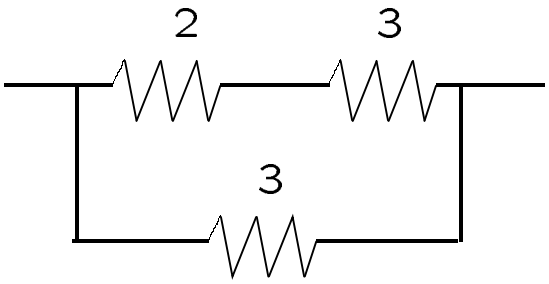


- Two resistors in series, with resistances R_1 and R_2, respectively.
- Total resistance of the circuit is $5 + 6 = 11$.
- Can represent the circuit as a function term: `series(5,6)`.

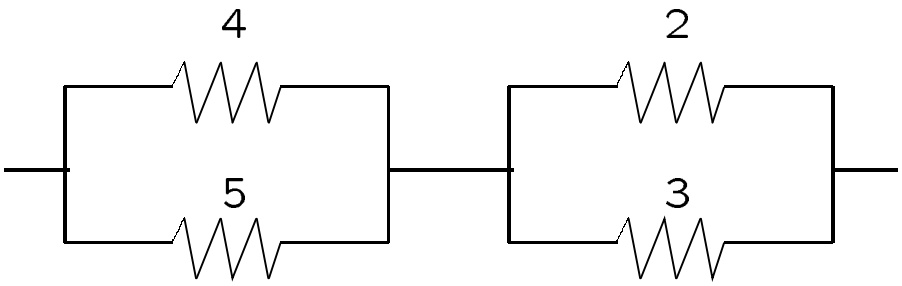


- Two resistors in parallel.
- Total resistance of the circuit is $\frac{2 \times 3}{2+3} = 1.2$
- Represent the circuit as a function term: `par(2,3)`.

More Complex Circuits




```
par(3, series(2,3))
```




```
series(par(4,5), par(2,3))
```

Problem:

Write a Prolog program that computes the total resistance of any circuit.

For example,

Query: `resistance(series(1,2), R)`
Answer: `R = 1+2 = 3`

Query: `resistance(par(2,3), R)`
Answer: `R = (2*3)/(2+3) = 6/5 = 1.2`

Query: `resistance(series(3,par(2,3)), R)`
Answer: `R = 3 + 1.2 = 4.2`

Query: `resistance(3, R)`
Answer: `R = 3`

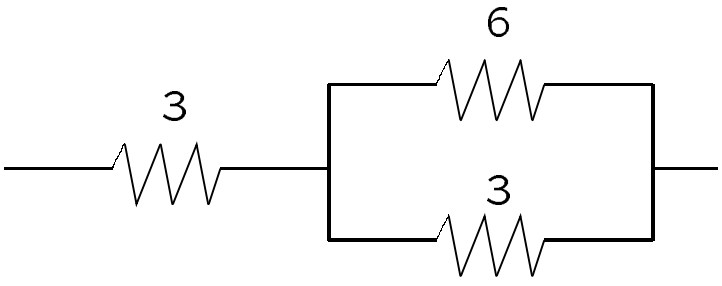
Solution

```
(1)   resistance(R,R) :- number(R).

(2)   resistance(series(C1,C2), R)
         :- resistance(C1, R1),
            resistance(C2, R2),
            R is R1+R2.

(3)   resistance(par(C1,C2), R)
         :- resistance(C1,R1),
            resistance(C2,R2),
            R is (R1*R2)/(R1+R2).
```

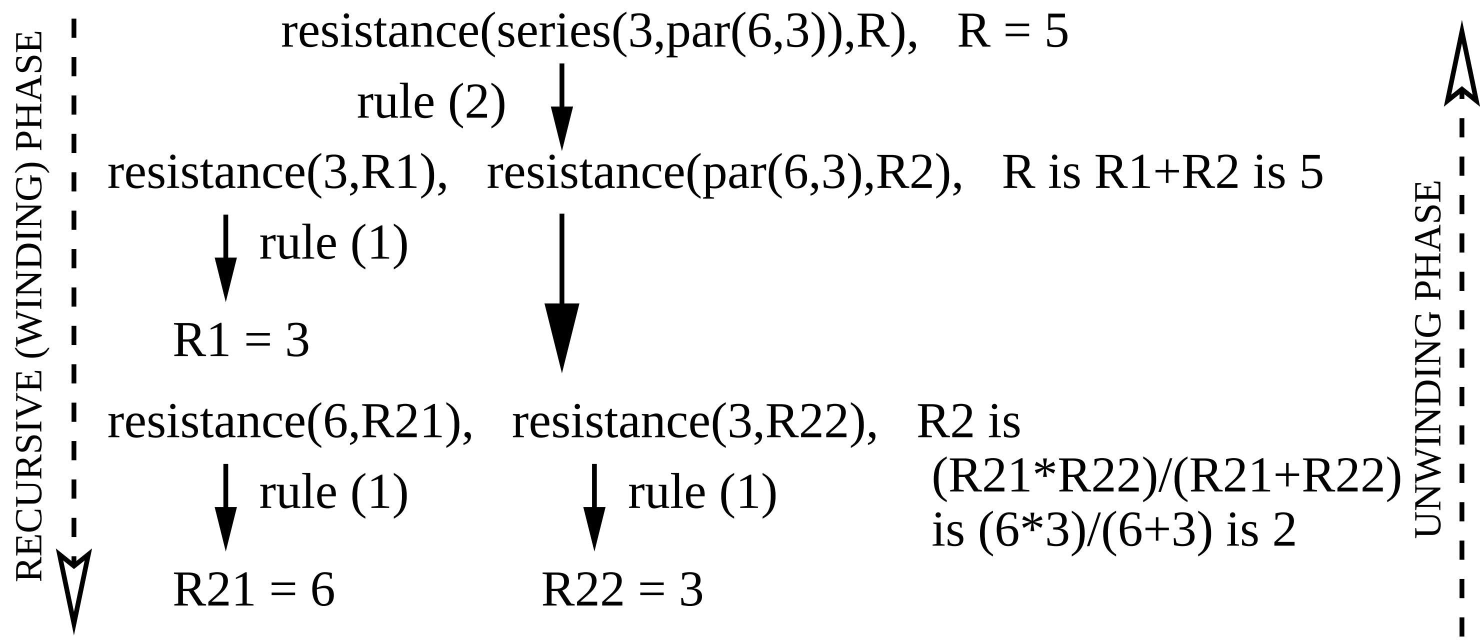
Sample Query:

```
resistance(series(3,par(6,3)), TR)
```

i.e., compute the total resistance, `TR`, of the following circuit:



a



RECURSIVE (WINDING) PHASE
resistance(series(3,par(6,3)),R),   R = 5
rule (2)
resistance(3,R1),   resistance(par(6,3),R2),   R is R1+R2 is 5
rule (1)
R1 = 3
resistance(6,R21),   resistance(3,R22),   R2 is
(R21*R22)/(R21+R22)
is (6*3)/(6+3) is 2
rule (1)
rule (1)
R21 = 6
R22 = 3
UNWINDING PHASE