Problem 1

For the following transition system,

\[ \begin{align*}
  s_1 & \xrightarrow{\{ a \}} s_2 \\
  s_2 & \xrightarrow{\{ a \}} s_3 \\
  s_3 & \xrightarrow{\{ a, b \}} s_4 \\
  s_4 & \xrightarrow{\{ b \}} s_1
\end{align*} \]

Determine which states satisfy each given LTL formula below:

(a) $\bigcirc a$
(b) $\bigcirc \bigcirc \bigcirc a$
(c) $\Box b$
(d) $\Box \Diamond a$
(e) $\Box (b \lor a)$
(f) $\Diamond (a \lor b)$
Problem 2

For the following transition system and the given formulas, determine which states satisfy each given formula:

\[ \Phi_1 = \forall (a \cup b) \lor \exists \bigcirc (\forall \square b) \]
\[ \Phi_2 = \forall \square \forall (a \cup b) \]
\[ \Phi_3 = (a \land b) \rightarrow \exists \bigcirc \exists \bigcirc \forall (b \lor a) \]
\[ \Phi_4 = (\forall \square \exists \bigcirc \Phi_3) \]
Problem 3

Consider the following three simple constraints about three unknown LTL formulas $F$, $G$, and $H$:

\[
\begin{align*}
F & \equiv a \lor G \\
G & \equiv b \land \Box F
\end{align*}
\]

Find (standard non-recursive) LTL formulas to stand for $F$ and $G$ above such that the constraints are satisfied and the formulas represent the smallest set of paths satisfying the constraints.

(a) $F \equiv$

(b) $G \equiv$

What if we are interested in the formulas representing the largest set of paths satisfying the constraints?

(a) $F \equiv$

(b) $G \equiv$