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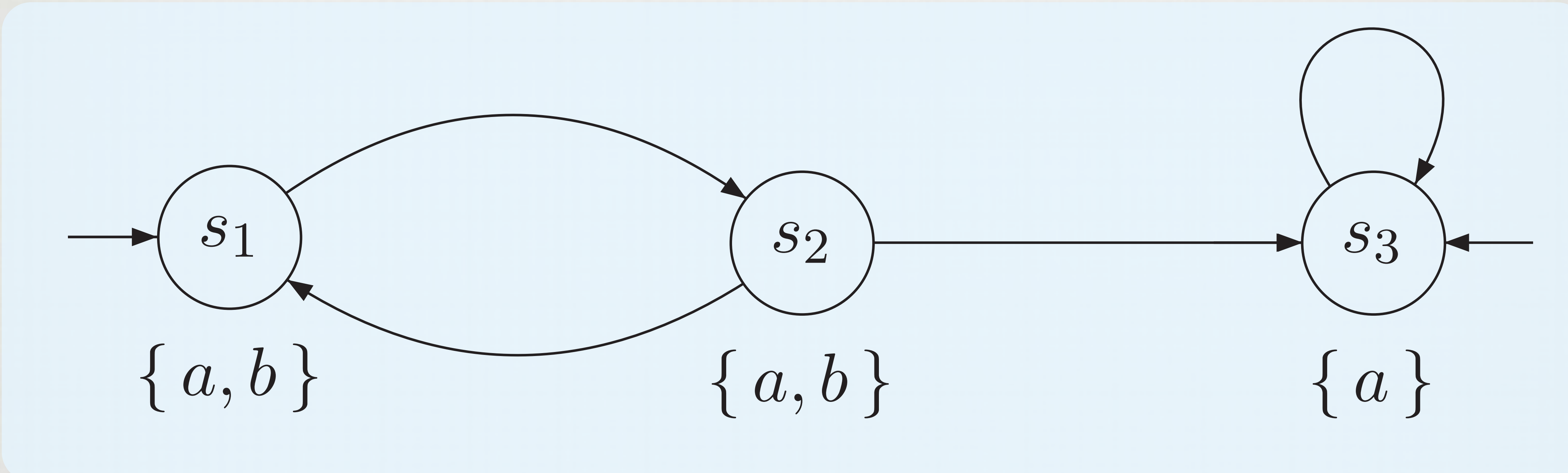
$$TS \models \phi \iff \forall s \in I : \; s \models \phi$$

# Examples



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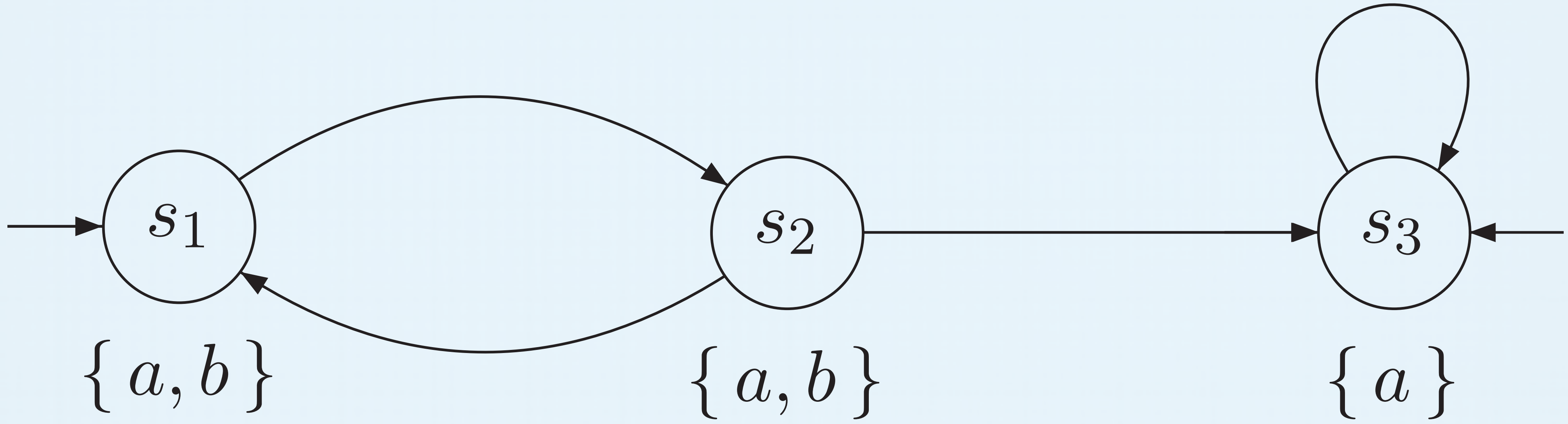


$$s_1 \models \bigcirc (a \wedge b)$$

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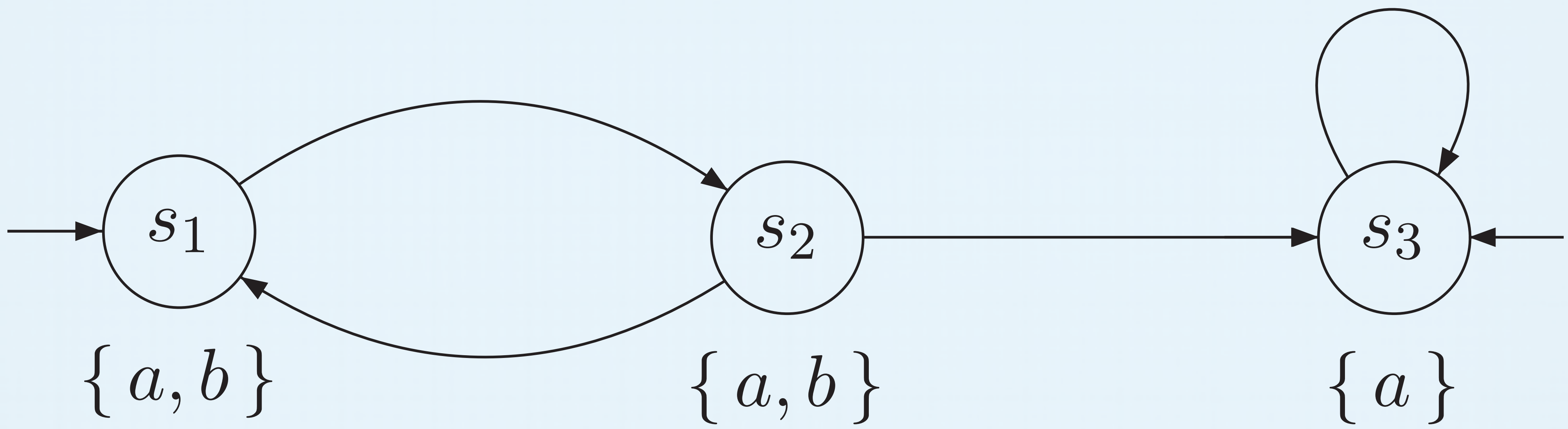


$$s_1 \models \bigcirc (a \wedge b)$$

$$s_2 \not\models \bigcirc (a \wedge b)$$

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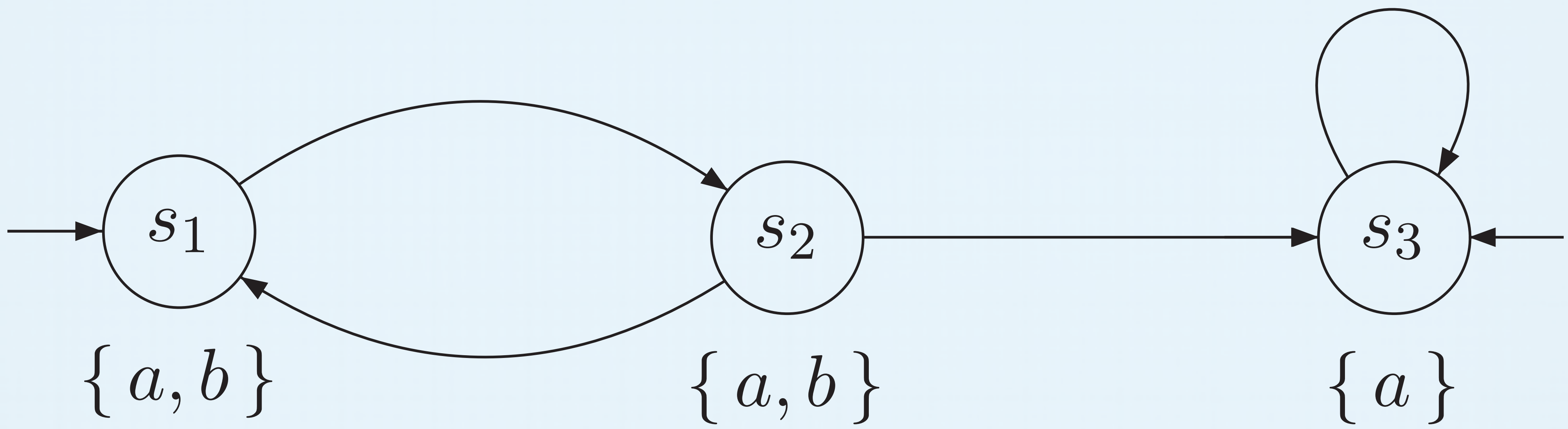
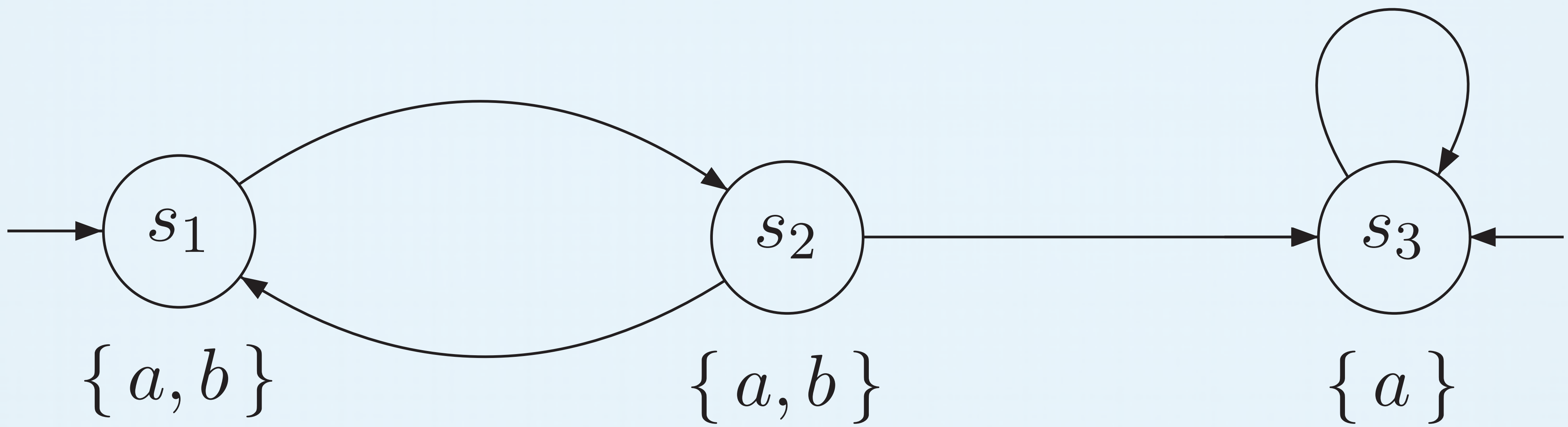



$$s_1 \models \bigcirc (a \wedge b)$$

$$s_2 \not\models \bigcirc (a \wedge b)$$

$$s_3 \not\models \bigcirc (a \wedge b)$$

# Examples

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# Negation?

$$\pi \models \phi \iff \pi \not\models \neg\phi$$

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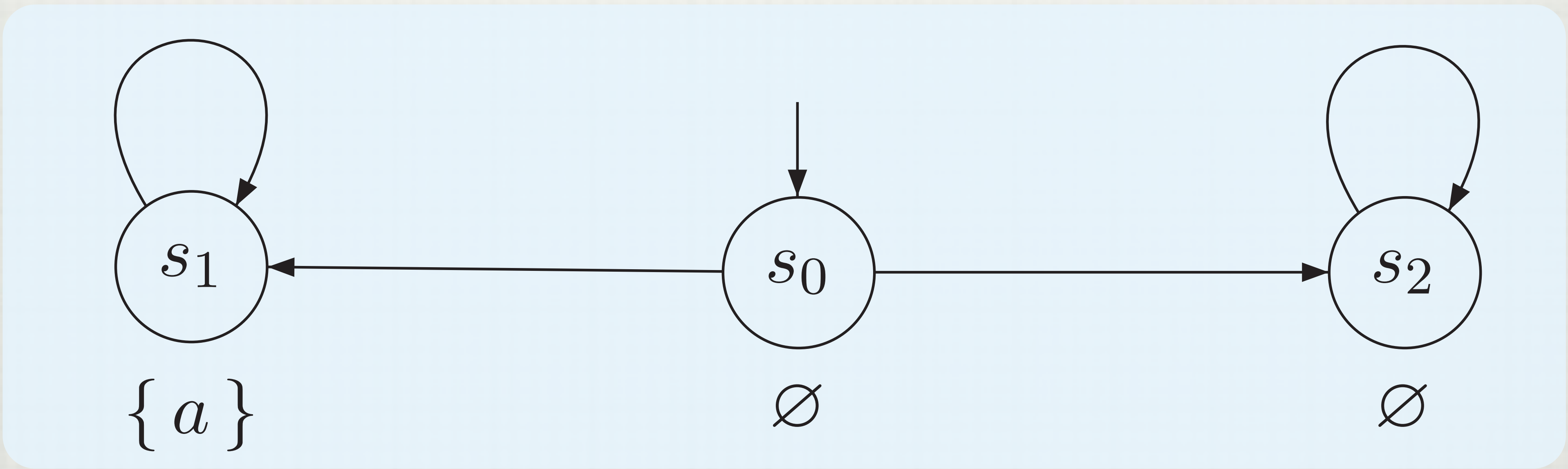

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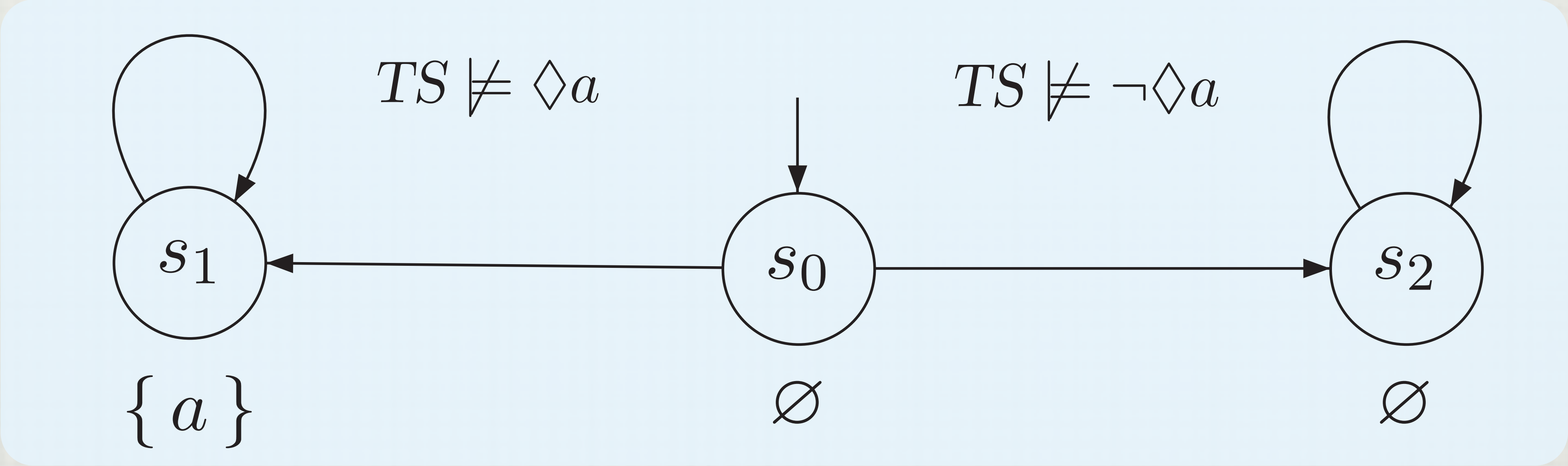

# Equivalence of LTL formulae

**Definition.** two LTL formulas are equivalent iff:

$$\forall \pi : \; \pi \models \phi_1 \iff \pi \models \phi_2$$

# Equivalence of LTL formulae

| | |
|---|---|
| *duality law*<br>$\neg \bigcirc \varphi \equiv \bigcirc \neg \varphi$<br>$\neg \Diamond \varphi \equiv \Box \neg \varphi$<br>$\neg \Box \varphi \equiv \Diamond \neg \varphi$ | *idempotency law*<br>$\Diamond \Diamond \varphi \equiv \Diamond \varphi$<br>$\Box \Box \varphi \equiv \Box \varphi$<br>$\varphi \, \mathsf{U} \, (\varphi \, \mathsf{U} \, \psi) \equiv \varphi \, \mathsf{U} \, \psi$<br>$(\varphi \, \mathsf{U} \, \psi) \, \mathsf{U} \, \psi \equiv \varphi \, \mathsf{U} \, \psi$ |
| *absorption law*<br>$\Diamond \Box \Diamond \varphi \equiv \Box \Diamond \varphi$<br>$\Box \Diamond \Box \varphi \equiv \Diamond \Box \varphi$ | *expansion law*<br>$\varphi \, \mathsf{U} \, \psi \equiv \psi \vee (\varphi \wedge \bigcirc (\varphi \, \mathsf{U} \, \psi))$<br>$\Diamond \psi \equiv \psi \vee \bigcirc \Diamond \psi$<br>$\Box \psi \equiv \psi \wedge \bigcirc \Box \psi$ |
| *distributive law*<br>$\bigcirc (\varphi \, \mathsf{U} \, \psi) \equiv (\bigcirc \varphi) \, \mathsf{U} \, (\bigcirc \psi)$<br>$\Diamond (\varphi \vee \psi) \equiv \Diamond \varphi \vee \Diamond \psi$<br>$\Box (\varphi \wedge \psi) \equiv \Box \varphi \wedge \Box \psi$ | |

# Expansion Laws

$$\varphi \,\mathsf{U}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

$$\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$$

$$\Box\psi \equiv \psi \wedge \bigcirc\Box\psi$$

# Expansion Laws

$$\varphi \,\mathsf{U}\, \psi \equiv \psi \lor (\varphi \land \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

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Lemma. Until is the least solution to the expansion law.

# Expansion Laws

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Lemma. Until is the least solution to the expansion law.

The following equation has many solutions:

$$X = \psi \vee (\phi \wedge \circ X)$$

Until is the smallest set that satisfies this equation.

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The following equation has many solutions:

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Until is the smallest set that satisfies this equation.

Note that we are using the notions of sets (of paths) and formulas interchangeably, by referring to the set of paths that satisfy a given formula.

# End of LTL!