The Exam

Problem 1: Model Checking

(20 points) Consider the labeled transition system below where each state is labeled with the set of atomic propositions that hold true in it.

Answer the two questions below, but keep in mind that a wrong answer carries a negative weight. For example, “correct answer 1, correct answer 2” will get more marks than “correct answer 1, correct answer 2, incorrect answer 1”. Otherwise, you can just list all states for every item, and you are guaranteed to include all correct answers! No justification is necessary. We will only mark the list of states.

(a) List the states that satisfy the LTL formula $\neg \Box (a \lor b)$.
   $q_0$

(b) List the states that satisfy the CTL formula $\exists (b \lor \forall (a \lor c))$.
   $q_2, q_3, q_4$
Problem 2: LTL

(35 points) For arbitrary LTL formulas $\phi$ and $\psi$:

(a) Prove or disprove $\phi \mathcal{U} (\phi \rightarrow \psi) \equiv (\phi \mathcal{U} \psi)$, with the appropriate formal justification.

Let $\phi = a$ and $\psi = b$ for atomic propositions $a$ and $b$. The following path is a counterexample to $\phi \mathcal{U} (\phi \rightarrow \psi) \Rightarrow (\phi \mathcal{U} \psi)$

$$\pi : \neg a \land \neg b \rightarrow \neg a \land \neg b \rightarrow \ldots \rightarrow \neg a \land \neg b \rightarrow \ldots$$

Since $\pi \models a \mathcal{U} (a \rightarrow b)$ and $\pi \not\models (a \mathcal{U} b)$.

(b) Prove or disprove $\Diamond \Box \phi \land \Diamond \Box \psi \equiv \Diamond \Box (\phi \land \psi)$, with the appropriate formal justification.

$\pi \models \Diamond \Box \phi \land \Diamond \Box \psi$ $\iff$
$\pi \models \Diamond \Box \phi \land \pi \models \Diamond \Box \psi$ $\iff$
$(\exists i : \forall j \geq i : \pi[j] \models \phi) \land (\exists k : \forall l \geq k : \pi[l] \models \psi)$ $\iff$
$\exists m : \forall j \geq m : \pi[j] \models \phi \land \pi[m] \models \psi$ $\iff$ $(\text{Let } m = \text{max}(i, k).)$
$\exists m : \forall j \geq m : \pi[j] \models \phi \land \psi$ $\iff$
$\pi \models \Diamond \Box (\phi \land \psi)$

Note: the above proof is ever so slightly sloppy, since we are short-circuiting both directions through the “(Let $m = \text{max}(i, k).)$”. We interpret this English sentence slightly differently going forward and backward. But, this level of sloppiness is fine, because anyone with mathematical maturity would immediately understand the precise proof in both directions. There exists no leap in reasoning.

Problem 3: CTL

(a) (15 points) Prove or disprove $\neg \forall (\phi \mathcal{U} \psi) \equiv \exists (\neg \psi \mathcal{U} \neg \phi)$, with the appropriate formal justification.

Let $\phi = a$ and $\psi = b$ for atomic propositions $a$ and $b$. The following transition system or the corresponding tree are both counterexamples to $\exists (\neg \psi \mathcal{U} \neg \phi) \Rightarrow \neg \forall (\phi \mathcal{U} \psi)$:

$$\neg a \land b$$

since it satisfies $\exists (\neg \psi \mathcal{U} \neg \phi) = \exists (\neg b \mathcal{U} \neg a)$ but not $\neg \forall (\phi \mathcal{U} \psi) = \neg \forall (a \mathcal{U} b)$
(b) (10 points) Formalize the following English specification as a CTL formula using the atomic propositions \{req, acc, den\}: “Every time the system receives a request, the following three choices are available to it: it can immediately deny it in the next step, immediately accept it in the next step, or ignore it. But, if ignored, no future acceptances or denials should be issued.”. Note that ignoring a request means that neither an acceptance or a denial is issued in the next step. Hence, you may not assume that \neg acc \rightarrow den and vice versa.

\[ \forall \Box (req \implies (\exists \Box acc \land \exists \Box den \land \forall \Box (\neg acc \land \neg den))) \]

Problem 4: Expansion Laws

(20 points) Consider the following three simple constraints about three unknown LTL formulas \(F\), \(G\), and \(H\):

\[
\begin{align*}
F &\equiv a \lor (\Box G \land \Box H) \\
G &\equiv b \land \Box F \\
H &\equiv c \land \Box F
\end{align*}
\]

Find (standard non-recursive) LTL formulas to stand for \(F\), \(G\), and \(H\) above such that the constraints are satisfied and the formulas represent the smallest set of paths satisfying the constraints.

(a) \(F \equiv (b \land c) \lor a\)

(a) \(G \equiv b \land (b \land c) \lor a\)

(b) \(H \equiv c \land (b \land c) \lor a\)