CSC410
SAT Solving
Inverted Lecture

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Conjunctive Normal Form

- $\big((p \vee (\neg q)) \wedge r \wedge ((\neg r) \vee p \vee q)\big)$
- $\big((\neg r) \vee p \vee q\big)$ (Only one clause.)
- $\big((\neg r) \wedge p \wedge q\big)$ (Three singleton clauses.)

Converting to CNF

1. Eliminate implication and equivalence.
   Replace $(\alpha \to \beta)$ by $((\neg\alpha) \vee \beta)$
   Replace $(\alpha \leftrightarrow \beta)$ by $((\neg\alpha) \vee \beta) \wedge (\alpha \vee (\neg\beta))$.
   (*Now only* $\wedge$, $\vee$ *and* $\neg$ *appear as connectives.*)
2. Apply De Morgan's and double-negation laws as often as possible.
   Replace $(\neg(\alpha \vee \beta))$ by $((\neg\alpha) \wedge (\neg\beta))$.
   Replace $(\neg(\alpha \wedge \beta))$ by $((\neg\alpha) \vee (\neg\beta))$.
   Replace $(\neg(\neg\alpha))$ by α.
   (*Now negation only occurs in literals.*)
3. Transform into a conjunction of clauses using distributivity.
   Replace $(\alpha \vee (\beta \wedge \gamma))$ by $((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$.
   (*One could stop here, but....*)
4. Simplify using idempotence, contradiction, excluded middle and Simplification I & II.

How do we decide if a propositional logic formula is satisfiable or unsatisfiable?

Example

$$p \wedge q \wedge (\neg p \vee \neg q)$$

Satisfiable or unsatisfiable?

Inference Rules

Premises

Conclusion

$$\frac{\alpha_1 \quad \alpha_2 \quad \ldots \quad \alpha_n}{\beta}$$

Examples:

$$\frac{\alpha \quad \beta}{(\alpha \wedge \beta)} \qquad \frac{(\alpha \wedge \beta)}{(\alpha \vee \beta)}$$

What is a proof?

Start with some premises and using the rules, derive a conclusion!

Binary Resolution Rule

$$\frac{(\alpha \vee p) \quad ((\neg p) \vee \beta)}{(\alpha \vee \beta)}$$

- It works with formulas in CNF forms.
- It is used to prove contradictions.

Example

$$p \land q \land (\neg p \lor \neg q)$$

Example

$$(p \lor q \lor r) \land (\neg p \lor q) \land (\neg q \lor \neg r)$$

Binary Resolution Rule in CDCL

$$\frac{(\alpha \vee p) \quad ((\neg p) \vee \beta)}{(\alpha \vee \beta)}$$

Unit resolution:

$$\frac{(\alpha \vee p) \quad (\neg p)}{\alpha}$$

Contradiction:

$$\frac{p \quad (\neg p)}{\perp}$$

And, the standard rule used in conflict clause learning.

Important Properties

Theorem. If from a set of clauses as a premise, we derive *false*, then the set of clauses is unsatisfiable.

Theorem. If there is no proof leading to *false*, then the set of clauses is satisfiable.