CSC410
Data Flow Analyses

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Properties of Transfer Functions

Monotonicity: $\forall x, y \in \mathbf{L} : x \sqsubseteq y \implies f(x) \sqsubseteq f(y).$

Transfer functions in a dataflow analysis must be monotone!

Distributivity: $\forall x, y \in \mathbf{L} : f(x \sqcap y) = f(x) \sqcap f(y).$

But not necessarily distributive!

Formally

For a path $\pi = init \ldots l$

Transfer Function for location l

$$f_\pi = f_{init} \circ \cdots \circ f_l$$

Set of all paths to l

Initial information at "init"

$$MOP_\circ(l) = \bigsqcap_{\pi \in Path(l)} f_\pi(\iota).$$

$$MOP_\bullet(l) = f_l(MOP_\circ(l)).$$

Good News

Instead, compute the *maximal fixed point solution* (MFP).

in the meet lattice

Consider the set of constraints below:

$$MFP_\circ(l) = \begin{cases} \iota & l = init \\ \prod_{(l',l)\in flow} MFP_\bullet(l') & \text{otherwise} \end{cases}$$

$$MFP_\bullet(l) = f_l(MFP_\circ(l))$$

The Coincidence

If transfer functions are monotone:

$$MOP_{\circ}(l) \sqsupseteq MFP_{\circ}(l) \qquad MOP_{\bullet}(l) \sqsupseteq MFP_{\bullet}(l)$$

Less Precise!

The fixpoint solution over-approximates the result!

If transfer functions are distributive:

$$MOP_{\circ}(l) = MFP_{\circ}(l) \qquad MOP_{\bullet}(l) = MFP_{\bullet}(l)$$

The two solutions coincide!

Proof of their relation

If transfer functions are monotone:

$$MOP_\circ(l) \sqsupseteq MFP_\circ(l) \qquad MOP_\bullet(l) \sqsupseteq MFP_\bullet(l)$$

(proof on the board)

Monotonicity: $\forall x, y \in \mathbf{L} : x \sqsubseteq y \implies f(x) \sqsubseteq f(y).$

alternatively: $\forall x, y \in \mathbf{L} : f(x \sqcap y) \sqsubseteq f(x) \sqcap f(y).$

(exercise for you: prove the two definitions are equivalent)