CSC410

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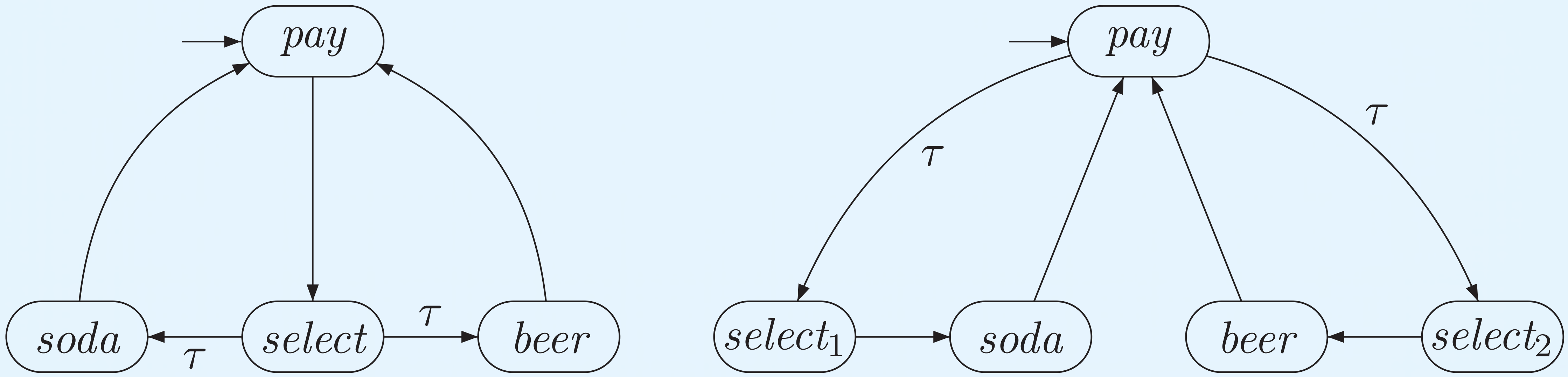
FALL 2020

Computational Tree Logic (CTL)

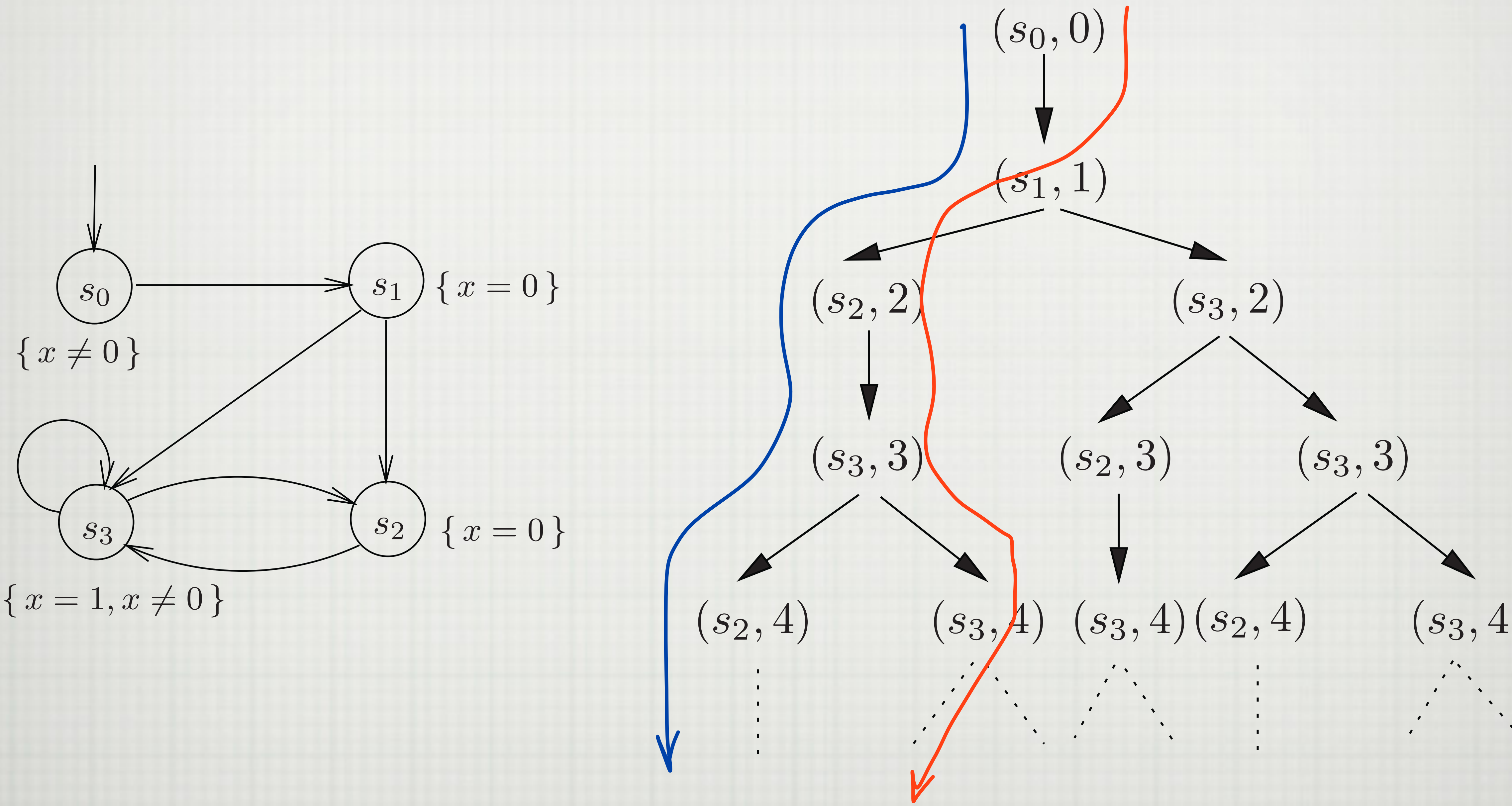
Limitations of LTL



They are trace equivalent.

These two transition systems satisfy the same set of LTL formulas. But they function in different ways.

Computational Tree Logic (CTL)



CTL Syntax

State Formula $\Phi ::= \text{true} \;\big|\; a \;\big|\; \Phi_1 \wedge \Phi_2 \;\big|\; \neg\Phi \;\big|\; \exists\varphi \;\big|\; \forall\varphi$

Path Formula $\varphi ::= \bigcirc\,\Phi \;\big|\; \Phi_1 \,\mathsf{U}\, \Phi_2$

Examples: $\exists\bigcirc(x=1)$ $\forall\bigcirc(x=1)$

But not: $\exists(x = 1 \;\wedge\; \forall\bigcirc(x \geqslant 3))$ $\exists\bigcirc(\text{true}\,\mathsf{U}\,(x=1))$

needs a path quantifier!

needs a temporal operator!

Eventually and Always

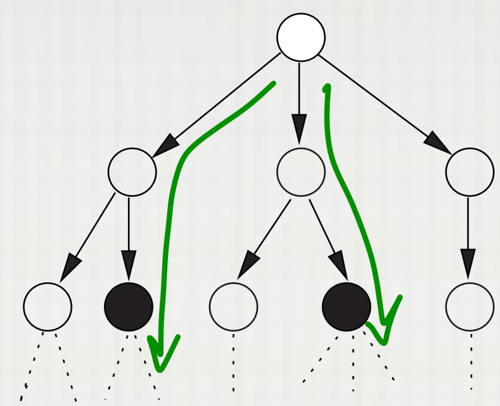
$$\text{eventually:} \quad \exists\Diamond\Phi \quad = \quad \exists(\text{true}\,\mathsf{U}\,\Phi)$$

$$\forall\Diamond\Phi \quad = \quad \forall(\text{true}\,\mathsf{U}\,\Phi)$$

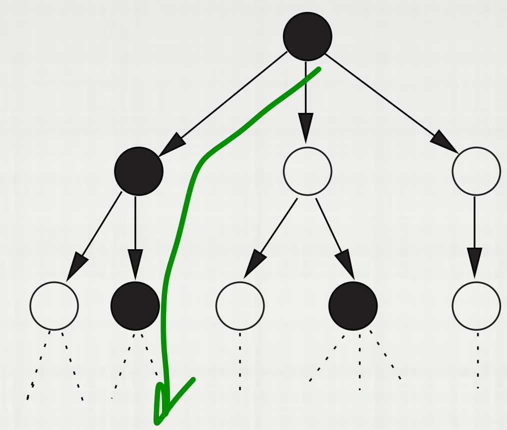
$$\text{always:} \quad \exists\Box\Phi \quad = \quad \neg\forall\Diamond\neg\Phi$$

$$\forall\Box\Phi \quad = \quad \neg\exists\Diamond\neg\Phi$$

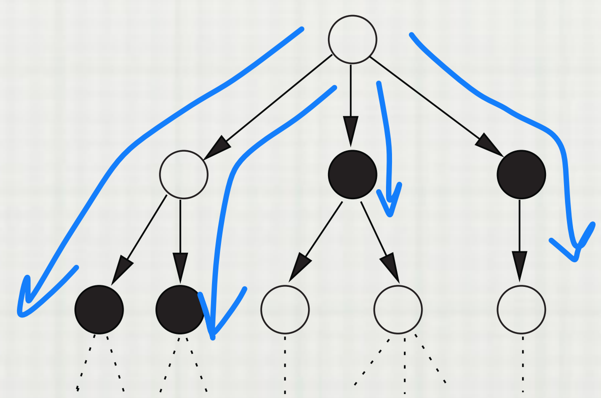
Meaning of CTL: Examples



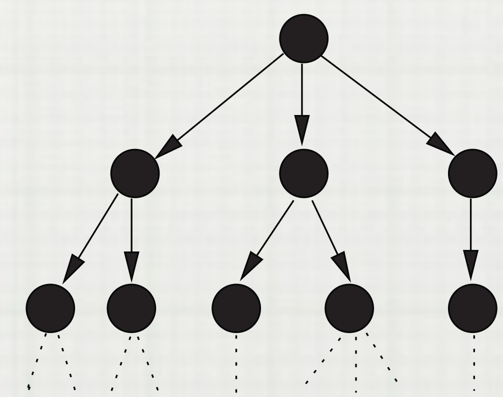
$\exists\Diamond$ *black*



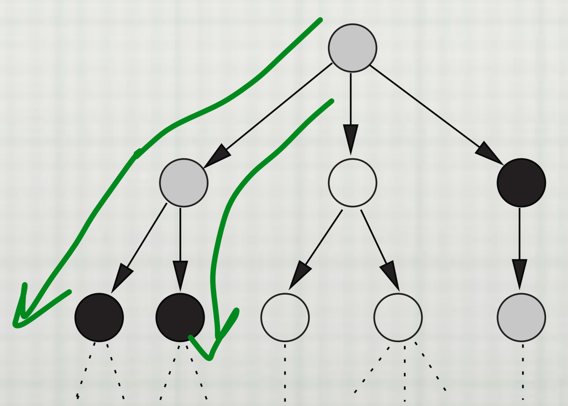
$\exists\Box$ *black*



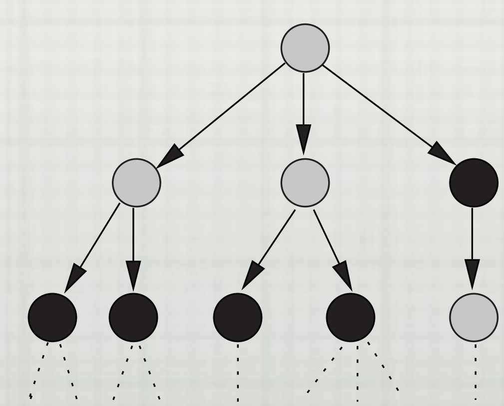
$\forall\Diamond$ *black*



$\forall\Box$ *black*



$\exists(gray \,\mathsf{U}\, black)$



$\forall(gray \,\mathsf{U}\, black)$

CTL Semantics

$$s \models a \quad \text{iff} \quad a \in L(s)$$ → atomic propositions

$$s \models \neg \Phi \quad \text{iff} \quad \text{not } s \models \Phi$$

$$s \models \Phi \wedge \Psi \quad \text{iff} \quad (s \models \Phi) \text{ and } (s \models \Psi)$$

$$s \models \exists \varphi \quad \text{iff} \quad \pi \models \varphi \text{ for some } \pi \in Paths(s)$$

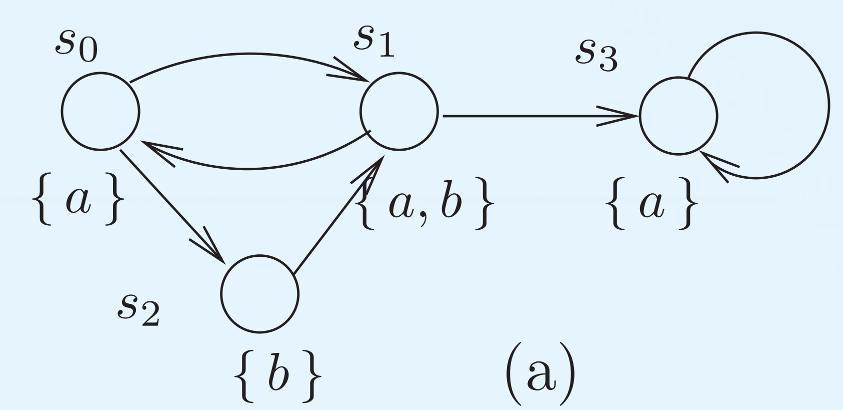
$$s \models \forall \varphi \quad \text{iff} \quad \pi \models \varphi \text{ for all } \pi \in Paths(s)$$

Same as LTL

$$\pi \models \bigcirc \Phi \quad \text{iff} \quad \pi[1] \models \Phi$$

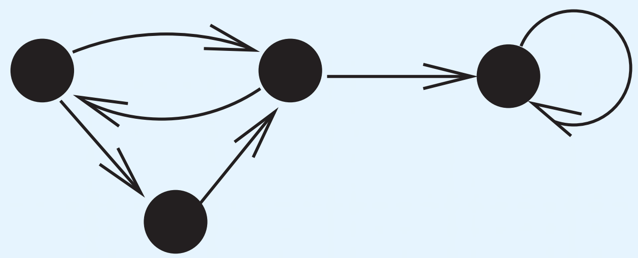
$$\pi \models \Phi \,\mathsf{U}\, \Psi \quad \text{iff} \quad \exists j \geqslant 0.\ (\pi[j] \models \Psi \ \wedge \ (\forall\, 0 \leqslant k < j.\, \pi[k] \models \Phi))$$

CTL for LTSs

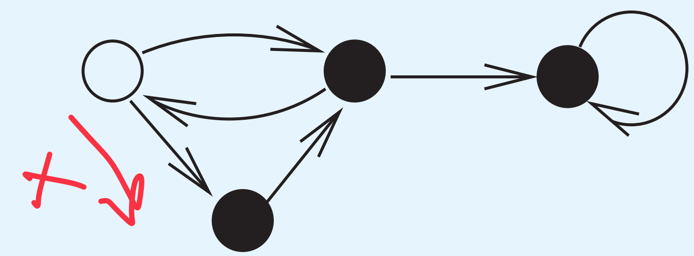


(a)

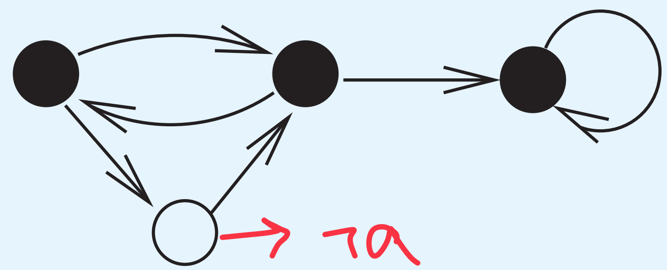
$\exists\bigcirc\, a$



$\forall\bigcirc\, a$

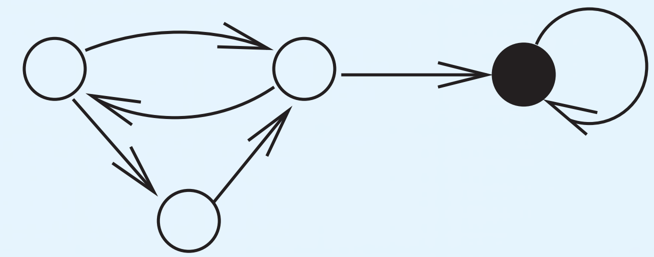


$\exists\Box\, a$

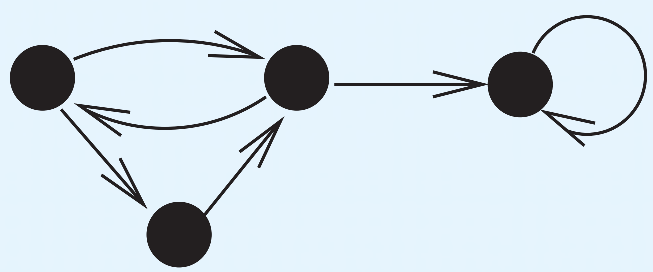


$\rightarrow \neg a$

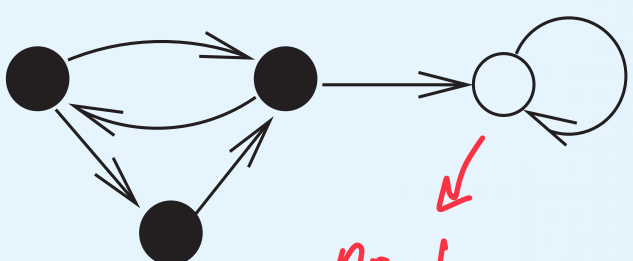
$\forall\Box\, a$



$\exists\Diamond\, \boxed{(\exists\Box\, a)}$

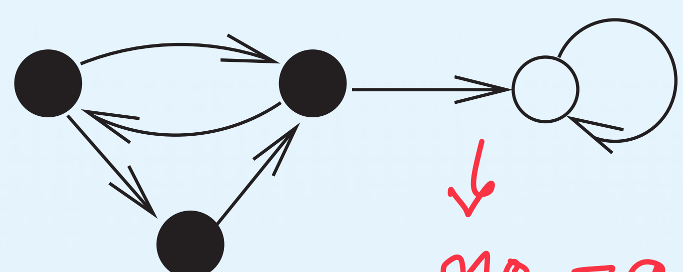


$\forall(a \,\mathsf{U}\, b)$



no $\underline{\underline{b}}$ in future!

$\exists\,(a \,\mathsf{U}\, (\neg a \;\wedge\; \forall(\neg a \,\mathsf{U}\, b)))$



no $\neg a$ in future!

End of CTL