Consider a formula $F$ that is being checked for satisfiability in CNF using the following set of clauses:

\[
\begin{align*}
    c_1 &= \neg x_2 \lor \neg x_4 \lor x_7 \\
    c_2 &= \neg x_2 \lor \neg x_7 \lor x_8 \\
    c_3 &= \neg x_5 \lor \neg x_8 \\
    c_4 &= x_4 \lor x_6 \\
    c_5 &= \neg x_1 \lor x_3 \lor x_5 \\
    c_6 &= x_2 \lor \neg x_5 \lor x_6 \\
\end{align*}
\]

Here is a hypothetical implication graph from decisions previously made at levels 3 and 5 of a run of the DPLL algorithm, and we add a current decision node at the current level 6.

The implication graph shows the relationships between clauses and variables. Each clause is represented by a node, and arrows indicate implications based on decisions made at different levels.

The clauses are:

- $c_1: \neg x_2 \lor \neg x_4 \lor x_7$
- $c_2: \neg x_2 \lor \neg x_7 \lor x_8$
- $c_3: \neg x_5 \lor \neg x_8$
- $c_4: x_4 \lor x_6$
- $c_5: \neg x_1 \lor x_3 \lor x_5$
- $c_6: x_2 \lor \neg x_5 \lor x_6$

The equation $c_7: \neg x_1 \lor x_3 \lor x_6$ asserts a contradiction, indicating that the formula is unsatisfiable.
Clause Learning

\[
\begin{align*}
  c_1 &= x_1 \lor x_{31} \lor \neg x_2 \\
  c_3 &= x_2 \lor x_3 \lor x_4 \\
  c_5 &= x_{21} \lor \neg x_4 \lor \neg x_6 \\
  c_2 &= x_1 \lor \neg x_3 \\
  c_4 &= \neg x_4 \lor \neg x_5 \\
  c_6 &= x_5 \lor x_6
\end{align*}
\]

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\[
\begin{align*}
  c_6 &: x_5 \lor x_6 \\
  \neg x_4 \lor x_6 \\
  \neg x_4 \lor x_{21} \\
  x_2 \lor x_3 \lor x_{21} \\
  x_1 \lor x_3 \lor x_{31} \lor x_{21} \\
  x_1 \lor x_{31} \lor x_{21}
\end{align*}
\]