Problem 2: DPLL Conflict Clause Learning

(50 points) Consider a formula $F$ that is being checked for satisfiability in CNF using the following set of clauses:

$$
\begin{align*}
    c_1 &= \neg x_2 \lor \neg x_4 \lor x_7 \\
    c_2 &= \neg x_2 \lor \neg x_7 \lor x_8 \\
    c_3 &= \neg x_5 \lor \neg x_8 \\
    c_4 &= x_4 \lor x_6 \\
    c_5 &= \neg x_1 \lor x_3 \lor x_5 \\
    c_6 &= x_2 \lor \neg x_5 \lor x_6
\end{align*}
$$

Consider a hypothetical situation where we inherit the following implication graph from decisions previously made at levels 3 and 5 of a DPLL algorithm, and we add a current decision node at the current level 6.

![Implication Graph](image)

(a) (15 points) Complete the above implication graph (in place) for level 6 to get a conflict.
(b) (35 points) Produce an asserting clause that would be learned by the DPLL algorithm before it backtracks. To justify your answer, write down the sequence of binary resolution steps (and all the intermediate clauses) that gets you to your asserting clause. For clarity, label each clause that you use with its identifier from the list, e.g. $c_5 : \neg x_1 \lor x_3 \lor x_5$ instead of just $\neg x_1 \lor x_3 \lor x_5$.

Hint: there is more than one correct answer for an asserting clause, but the shortest one has only two literals. We will accept any asserting clause as a correct answer.