Tutorial Week 6: SMT
Garden Tree Problem

• Given a garden with 5×5 slots for trees

• An infinite number of trees with heights: 1, 2, 3, 4

• Two trees with the same height \( x \) cannot be placed within \( x \) radius

• We want to find an arrangement of trees to maximizing the total height
Garden Example

3 - 1 4 3
- 1 - 2 1
1 2 - 1 -
4 1 - - -
1 3 - 2 4

Total Height: 34

Can we do better?

We can solve a simpler problem first
Simplified Garden Tree Problem

• Given a garden with $N \times N$ slots

• A finite multi-set of trees of different heights: \{h1:8, h2: 4, h3:3, h4:3\}

• Two trees with the same height $x$ cannot be placed within $x$ radius

- 3 2 4 1
- 1 2
2 1 1 3
4 1 1
1 3 2 1 4

Total Height: 37
Simplified Garden Tree Problem Constraints

Each tree has three symbolic attributes:

\[ x: \text{nat} \quad \text{the x coordinate of the tree} \]
\[ y: \text{nat} \quad \text{the y coordinate of the tree} \]

Constraints:

1. Range constraints for trees’ coordinates
2. Every slot can only contain one tree
3. Trees with the same height cannot be placed within a certain radius
Challenge: Multi-set of Trees

• A finite multi-set of trees of different heights: \{h1:8, h2: 4, h3:3, h4:3\}

The multi-set is currently given by us, how do we identify a multi-set of trees such that the total height is better than what we already have?

We can use the upper-bound of the multi-set as the search space:

\{h1:13, h2: 5, h3:4, h4:3\}

and search for a multi-set within the search space ....
Garden Tree Problem Constraints

Each tree has three symbolic attributes:

- $planted: bool$ If the tree is planted
- $x: nat$ the x coordinate of the tree
- $y: nat$ the y coordinate of the tree

Constraints:
1. Range constraints for trees’ coordinates
2. Every slot can only plant one tree
3. Planted trees with the same height cannot be placed within a certain radius
4. The total height of the planted trees must be no less than the target value
Garden Tree

2 - 1 3 1
3 1 2 1 4
1 - 1 - 2
2 1 - 1 3
4 3 1 - 1

Total Height: 38

\{h1:10, h2: 4, h3:4, h4:2\}

Can we find the optimal solution?

Linear search by increasing the target value until UNSAT!