Tutorial Week 6: SMT

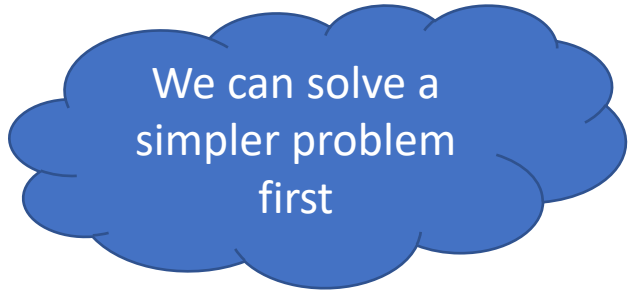
Garden Tree Problem

- Given a garden with 5×5 slots for trees
- An infinite number of trees with heights: 1, 2, 3, 4
- Two trees with the same height x cannot be placed within x radius
- We want to find an arrangement of trees to maximizing the total height

Garden Example

3 - 1 4 3
- 1 - 2 1
1 2 - 1 -
4 1 - - -
1 3 - 2 4

Total Height: 34

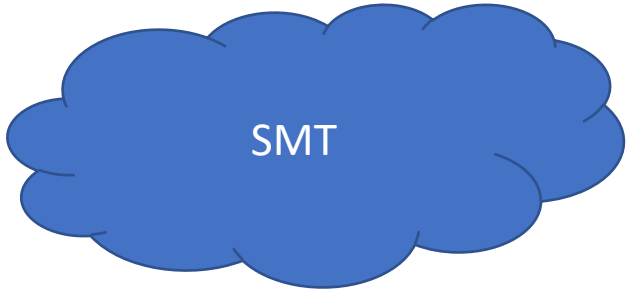


Simplified Garden Tree Problem

- Given a garden with $N \times N$ slots
- A finite multi-set of trees of different heights: {h1:8, h2: 4, h3:3, h4:3}
- Two trees with the same height x cannot be placed within x radius

- 3 2 4 1
- - 1 - 2
2 1 - 1 3
4 - 1 - 1
1 3 2 1 4

Total Height: 37



Simplified Garden Tree Problem Constraints

Each tree has three symbolic attributes:

$x: nat$	the x coordinate of the tree
$y: nat$	the y coordinate of the tree

Constraints:

1. Range constraints for trees' coordinates
2. Every slot can only contain one tree
3. Trees with the same height cannot be placed within a certain radius

Challenge: Multi-set of Trees

- A finite multi-set of trees of different heights: {h1:8, h2: 4, h3:3, h4:3}

The multi-set is currently given by us, how do we identify a multi-set of trees such that the total height is better than what we already have?

We can use the **upper-bound** of the multi-set as the search space:

{h1:13, h2: 5, h3:4, h4:3}

and search for a multi-set within the search space

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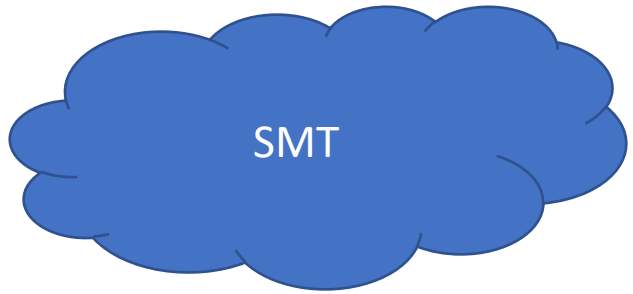

Garden Tree Problem Constraints

Each tree has three symbolic attributes:

$planted: bool$	If the tree is planted
$x: nat$	the x coordinate of the tree
$y: nat$	the y coordinate of the tree

Constraints:

1. Range constraints for trees' coordinates
2. Every slot can only plant one tree
3. Planted trees with the same height cannot be placed within a certain radius
4. The total height of the planted trees must be no less than the target value

Garden Tree

2 - 1 3 1
3 1 2 1 4
1 - 1 - 2
2 1 - 1 3
4 3 1 - 1

Total Height: 38

{h1:10, h2: 4, h3:4, h4:2}

Linear search by increasing the target value until UNSAT!

Can we find the optimal solution?