## Decisions with Multiple Agents: Game Theory

Alice Gao Lecture 23 Readings: RN 17.5. PM 11.1 - 11.3.

#### Outline

#### Learning Goals

#### Dancing or Running? — Pareto Optimality

Prisoner's dilemma

Matching Quarters — Mixed-Strategy Nash Equilibrium

Dancing or concert?

Revisiting the Learning goals

By the end of the lecture, you should be able to

- Determine Pareto optimal outcomes of a 2-player normal form game.
- Calculate a mixed strategy Nash equilibrium of a 2-player normal form game.

Learning Goals

#### Dancing or Running? — Pareto Optimality

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## Pareto dominance and Pareto optimality

#### Pareto dominance:

An outcome o Pareto dominates another outcome o' if and only if every player is weakly better off in o and at least one player is strictly better off in o.

#### A Pareto optimal outcome:

An outcome o is Pareto optimal if and only if no other outcome o' Pareto dominates o.

What is the difference between the statements below?

- An outcome *o* Pareto dominates all other outcomes.
- An outcome o is NOT Pareto dominated by any other outcome.

CQ: Dancing or running - Pareto optimality

#### CQ: How many of the four outcomes are Pareto optimal?

Bob

		dancing	running
Alice	dancing	(2, 2)	(0, 0)
	running	(0,0)	(1, 1)

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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Matching Quarters — Mixed-Strategy Nash Equilibrium

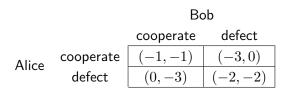
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## Prisoner's dilemma

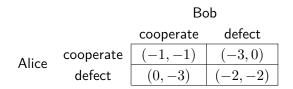


Alice and Bob have been caught by the police. Each has been offered a deal to testify against the other. They had originally agreed not to testify against each other. However, since this agreement cannot be enforced, each must choose whether to honour it. If both refuse to testify, both will be convicted of a minor charge due to lack of evidence and serve 1 year in prison. If only one testifies, the defector will go free and the other one will be convicted of a serious charge and serve 3 years in prison. If both testify, both will be convicted of a major charge and serve 2 years in prison.

## CQ: Prisoner's dilemma - DSE

CQ: Which outcome, if any, is a dominant strategy equilibrium?

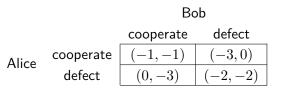
- (A) (cooperate, cooperate)
- (B) (cooperate, defect)
- (C) (defect, cooperate)
- (D) (defect, defect)
- (E) There is no dominant strategy equilibrium.



### CQ: Prisoner's dilemma - NE

## **CQ:** How many of the four outcomes are **pure-strategy Nash equilibria**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4



CQ: Prisoner's dilemma - Pareto optimality

CQ: How many of the four outcomes are Pareto optimal?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 Bob Alice  $\begin{array}{c} \text{Bob}\\ \text{cooperate} & \text{defect}\\ \text{defect} & (-1,-1) & (-3,0)\\ (0,-3) & (-2,-2) \end{array}$  Learning Goals

Dancing or Running? — Pareto Optimality

Prisoner's dilemma

#### Matching Quarters — Mixed-Strategy Nash Equilibrium

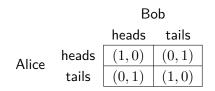
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## Matching quarters

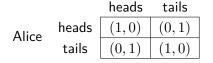


Alice and Bob are playing the game of matching quarters. They each show one side of a quarter. Alice wants the sides of the two quarters to match, whereas Bob wants the sides of the two quarters to NOT match. CQ: Matching quarters - Pure-Strategy NE

# **CQ:** How many of the four outcomes are **pure-strategy Nash equilibria**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Bob



Matching quarters - Mixed-Strategy NE

How do we calculate a mixed-strategy Nash equilibrium?

What does it mean if a player is mixing between two actions?

Learning Goals

Dancing or Running? — Pareto Optimality

Prisoner's dilemma

Matching Quarters — Mixed-Strategy Nash Equilibrium

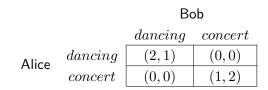
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## Dancing or concert?



Alice and Bob want to sign up for an activity together. They both prefer to sign up for the same activity. However, Alice prefers dancing over going to a concert whereas Bob prefers going to a concert over dancing. CQ: Why is a player willing to mix between two actions?

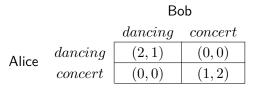
Consider a 2-player normal form game and fix Bob's strategy. Alice is willing to play heads 60% of the time and tails 40% of the time. Which of the following statements is true?

- (A) Alice's expected utility of playing heads is greater than her expected utility of playing tails.
- (B) Alice's expected utility of playing heads is less than her expected utility of playing tails.
- (C) Alice's expected utility of playing heads is same as her expected utility of playing tails.

## CQ: Dancing or concert - mixed-strategy NE

**CQ:** At the mixed strategy Nash equilibrium, with what probability does **Alice go dancing**?

(A) 0 (B) 1/3 (C) 2/3 (D) 1

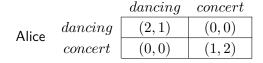


## CQ: Dancing or concert - mixed-strategy NE

**CQ:** At the mixed strategy Nash equilibrium, with what probability does **Bob go dancing**?

(A) 0 (B) 1/3 (C) 2/3 (D) 1

Bob



## Revisiting the Learning Goals

By the end of the lecture, you should be able to

- Determine Pareto optimal outcomes of a 2-player normal form game.
- Calculate a mixed strategy Nash equilibrium of a 2-player normal form game.