Decisions with Multiple Agents: Game Theory

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Lecture 22

Readings: RN 17.5. PM 11.1 - 11.3.

Outline

Learning Goals

By the end of the lecture, you should be able to

- Determine dominant-strategy equilibria of a 2-player normal form game.
- Determine pure-strategy Nash equilibria of a 2-player normal form game.

CQ: Prior knowledge w/ GT and MD

CQ: Have you learned Game Theory and/or Mechanism Design in another course?

(A) Yes

(B) No

Decision making with multiple agents

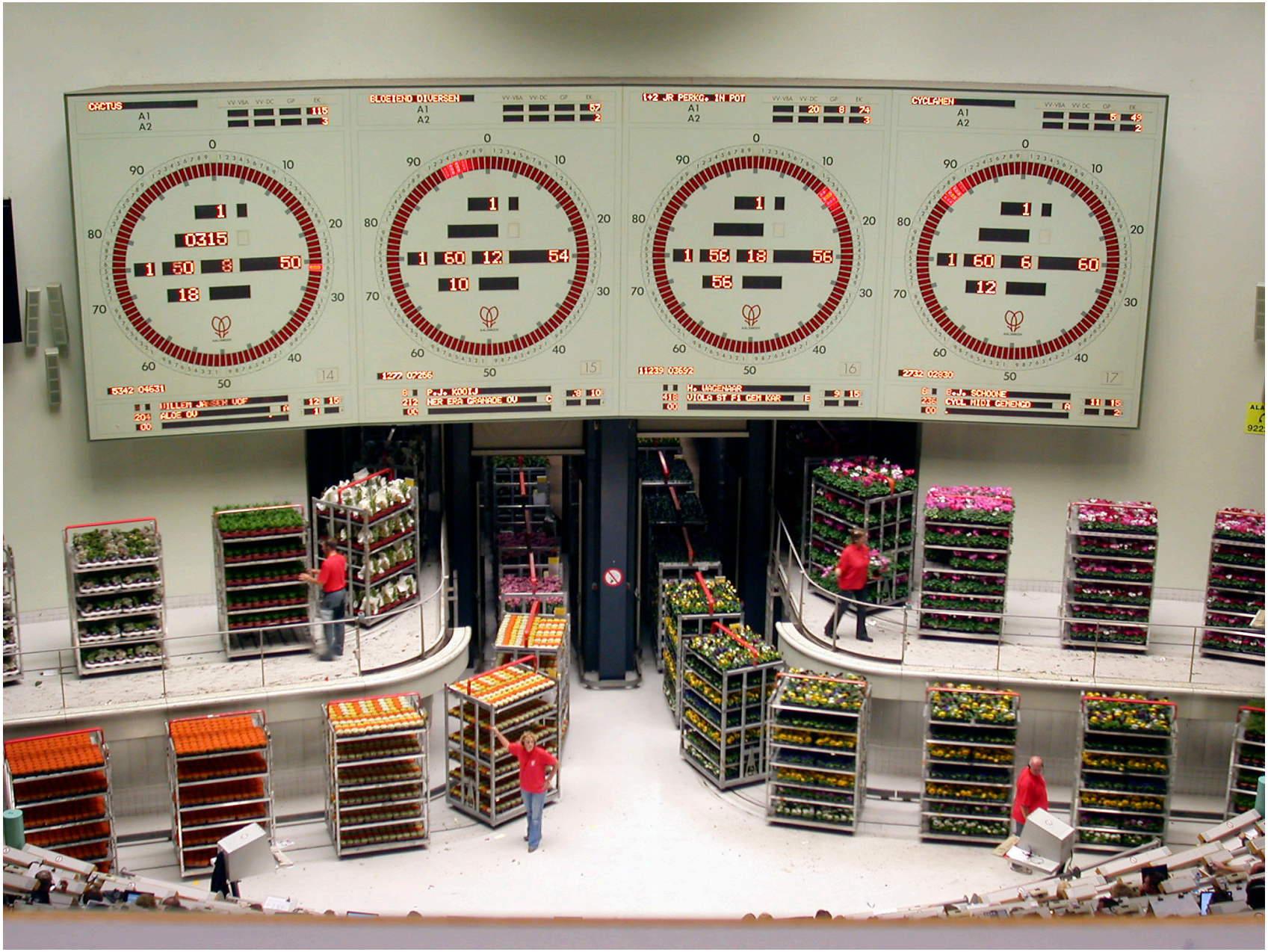
- Decision making in a multi-agent environment.
- When making a decision, each agent needs to take into account of the other agents' behaviour.

What is a game?

Game Theory

A game is a mathematical model of a strategic scenario.

Dutch flower auction



Matching problems

Examples: medical residency matching, school choice, and organ transplant, etc.

Crowdsourcing

Examples: 99 Designs, Topcoder, Duolingo, uwflow.com

Game Theory v.s. Mechanism Design

- Game theory:
  Given a game, how would agents play it?

- Mechanism design:
  How should we design the rules of the game so that the agents will behave the way we want them to?

The multi-agent framework

- Each agent decides what to do based on
  - their information about the world.
  - their information about other agents.
  - their utility function.
- The outcome depends on the actions of all agents.

Relationship between utility functions

A game can be

- cooperative where agents have a common goal.
- competitive where agents have conflicting goals.
- or somewhere in between.

Home or dancing?

		Bob: *home*	Bob: *dancing*
Alice: *home*		$(0, 0)$	$(0, 1)$
Alice: *dancing*		$(1, 0)$	$(2, 2)$

Alice and Bob are best friends in grad school. They both enjoy each other's company, but neither can communicate with the other before deciding whether to stay at home (where they would not see each other) or go swing dancing this evening (where they could see each other). Each prefers going dancing to being at home.

A Normal-Form Game

A normal-form game consists of

- A set of players and a set of actions for each player.
- Each outcome (e.g. $(home, dancing)$) consists of an action for each player.
- A payoff matrix.
  For each outcome, the utility pair (u_r, u_c) specifies that for this outcome, the row player has utility u_r and the column player has utility u_c.

Playing a Normal-Form Game

Players choose their actions at the same time,
without communicating with each other,
and without observing other players' actions.

Each player i chooses a mixed strategy σ_i.

- A mixed strategy is a probability distribution over the actions.
  e.g. stay at home w/ prob 80% go dancing w/ prob 20%.
- A pure strategy plays one action w/ probability 1.

A solution to a normal-form game is a strategy profile,
consisting of a mixed strategy for each player.

CQ: Home or dancing?
What do you think the players will do?

		Bob: *home*	Bob: *dancing*
Alice: *home*		$(0, 0)$	$(0, 1)$
Alice: *dancing*		$(1, 0)$	$(2, 2)$

(A) (home, home)

(B) (home, dancing)

(C) (dancing, home)

(D) (dancing, dancing)

Terminologies for Strategies and Utilities

Terminologies for strategies:

- σ_i denotes the strategy of player i.
- σ_{-i} denotes the strategies of all the players except i.

Terminologies for utilities:

- $U_i(\sigma) = U_i(\sigma_i, \sigma_{-i})$ denote the utility of agent i under the strategy profile σ.

Dominant Strategy Equilibrium

For player i, **a strategy σ_i dominates strategy σ_i'** iff

- $U_i(\sigma_i, \sigma_{-i}) \geq U_i(\sigma_i', \sigma_{-i}), \forall \sigma_{-i}$, and
- $U_i(\sigma_i, \sigma_{-i}) > U_i(\sigma_i', \sigma_{-i}), \exists \sigma_{-i}$

A dominant strategy dominates all other strategies.

When each player has a dominant strategy, the combination of those strategies is called **a dominant strategy equilibrium**.

CQ: Home or dancing - DSE

CQ: Which of the following statements is correct?

		Bob: *home*	Bob: *dancing*
Alice	*home*	$(0, 0)$	$(0, 1)$
	dancing	$(1, 0)$	$(2, 2)$

(A) (home, home) is the only dominant strategy equilibrium.

(B) (dancing, dancing) is the only dominant strategy equilibrium.

(C) (dancing, home) or (home, dancing) is the only dominant strategy equilibrium.

(D) This game has more than one dominant strategy equilibrium.

(E) This game has no dominant strategy equilibrium.

Dancing or running?

		Bob	
		$dancing$	$running$
Alice	$dancing$	$(2, 2)$	$(0, 0)$
	$running$	$(0, 0)$	$(1, 1)$

Alice and Bob would like to sign up for an activity together.
They both prefer dancing over running.
They also prefer signing up for the same activity over signing up for two different activities.

CQ: Dancing or running
What do you think the players will do?

		Bob: *dancing*	Bob: *running*
Alice	*dancing*	$(2, 2)$	$(0, 0)$
	running	$(0, 0)$	$(1, 1)$

(A) (dancing, dancing)

(B) (dancing, running)

(C) (running, dancing)

(D) (running, running)

CQ: Dancing or running - DSE

CQ: Which of the following statements is correct?

		Bob: *dancing*	Bob: *running*
Alice	*dancing*	$(2, 2)$	$(0, 0)$
	running	$(0, 0)$	$(1, 1)$

(A) (dancing, dancing) is the only dominant strategy equilibrium.

(B) (running, running) is the only dominant strategy equilibrium.

(C) (dancing, running) or (running, dancing) is the only dominant strategy equilibrium.

(D) This game has more than one dominant strategy equilibrium.

(E) This game has no dominant strategy equilibrium.

Nash Equilibrium

- Won Nobel prize in Economics.
- 1-page paper on Nash equilibrium and 26-page PhD thesis.
- Every finite game has at least one Nash equilibrium.

Best Response

Given a strategy profile (σ_i, σ_{-i}), agent i's strategy σ_i is a best response to the other agents' strategies σ_{-i} if and only if

$$U_i(\sigma_i, \sigma_{-i}) \geq U_i(\sigma_i', \sigma_{-i}), \forall \sigma_i' \neq \sigma_i.$$

Nash Equilibrium

A strategy profile σ is a Nash equilibrium if and only if each agent i's strategy σ_i is a best response to the other agents' strategies σ_{-i}.

CQ: Dancing or running - NE

CQ: Which of the following is correct?
Consider only pure-strategy Nash equilibria.

		Bob: *dancing*	Bob: *running*
Alice	*dancing*	$(2, 2)$	$(0, 0)$
	running	$(0, 0)$	$(1, 1)$

(A) (dancing, dancing) is the only Nash equilibrium.

(B) (running, running) is the only Nash equilibrium.

(C) (dancing, dancing) and (running, running) are both Nash equilibria.

(D) This game has more than two Nash equilibria.

Revisiting the Learning Goals

By the end of the lecture, you should be able to

- Determine dominant-strategy equilibria of a 2-player normal form game.
- Determine pure-strategy Nash equilibria of a 2-player normal form game.