Decision Networks

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Lecture 17

Readings: RN 16.5 - 16.6. PM 9.3 - 9.4.

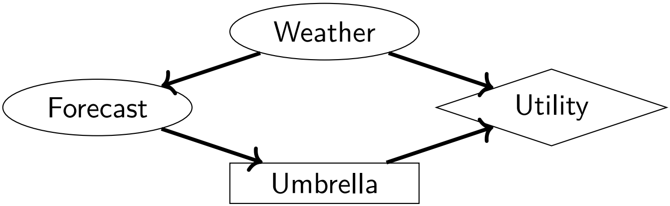
Outline

Learning Goals

By the end of the lecture, you should be able to

- Given a decision network with a single decision, determine the optimal policy and the expected utility of the optimal policy by enumerating all the policies.
- Given a decision network with a single decision, determine the optimal policy and the expected utility of the optimal policy by applying the variable elimination algorithm.
- Given a decision network with sequential decisions, determine the optimal policy and the expected utility of the optimal policy by applying the variable elimination algorithm.

The Weather Decision Network



The conditional probabilities

The Weather node:

Weather	P(Weather)
norain	0.7
rain	0.3

The Forecast node:

Weather	Forecast	P(Forecast | Weather)
norain	sunny	0.7
norain	cloudy	0.2
norain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

The utility function

The Utility node:

Weather	Umbrella	u(Weather, Umbrella)
norain	takeit	20
norain	leaveit	100
rain	takeit	70
rain	leaveit	0

A policy

- A policy specifies what the agent should do under all contingencies.
- For each decision variable, a policy specifies a value for the decision variable for each assignment of values to its parents.

For the weather decision network,
how many possible policies are there?

Solving the weather problem

Two approaches

- Compute the expected utility of each policy, and choose the policy that maximizes the expected utility.
- Use the variable elimination algorithm.

The Expected Utility of a Policy

Consider the policy π_1 below.

- take the umbrella if the forecast is cloudy, and
- leave the umbrella at home otherwise.

What is the expected utility of the policy π_1?

CQ: The Expected Utility of a Policy

CQ: Consider the policy π_2 below:

- take the umbrella if the forecast is rainy, and
- leave the umbrella at home otherwise.

What is the expected utility of the policy π_2?

Variable elimination algorithm

1. Remove all variables that are not ancestors of the utility node.
2. Define a factor for every non-decision node.
3. While there are decision nodes remaining
   3.1 Eliminate each random variable
   that is not a parent of a decision node.
   3.2 Find the optimal policy for the last decision and
   eliminate the decision variable.
4. Return the optimal policies.
5. Determine agent's expected utility following the optimal policy
   by eliminating all the remaining random variables.

Applying VEA: Step 1

Step 1: Remove all variables that are not ancestors of the utility node.

Every variable is an ancestor of the utility node.
There's nothing to be done.

Applying VEA: Step 2

Step 2: Define three factors f_1 for Weather, f_2 for Forecast, and f_3 for the Utility.

W	value
norain	0.7
rain	0.3

Table: Factor $f_1(W)$

W	F	value
norain	sunny	0.7
norain	cloudy	0.2
norain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

Table: Factor $f_2(W, F)$

Applying VEA: Step 2

Step 2: Define three factors f_1 for Weather, f_2 for Forecast, and f_3 for the Utility (continued).

W	U	value
norain	takeit	20
norain	leaveit	100
rain	takeit	70
rain	leaveit	0

Table: Factor $f_3(W, U)$

Applying VEA: Step 3.1

Step 3.1: Weather is not a parent of any decision node.

Eliminate Weather.
Multiply all the factors that contain Weather.

$$f_1(W) \times f_2(W, F) \times f_3(W, U) = f_4(W, F, U)$$

Sum out Weather from $f_4(W, F, U)$.

$$\sum_W f_4(W, F, U) = f_5(F, U)$$

Applying VEA: Step 3.1 Factor f_4

W	F	U	value
norain	sunny	takeit	9.8
norain	sunny	leaveit	49
norain	cloudy	takeit	2.8
norain	cloudy	leaveit	14
norain	rainy	takeit	1.4
norain	rainy	leaveit	7
rain	sunny	takeit	3.15
rain	sunny	leaveit	0
rain	cloudy	takeit	5.25
rain	cloudy	leaveit	0
rain	rainy	takeit	12.6
rain	rainy	leaveit	0

Table: Factor $f_4(W, F, U)$

Applying VEA: Step 3.1 Factor f_5

F	U	value
sunny	takeit	12.95
sunny	leaveit	49
cloudy	takeit	8.05
cloudy	leaveit	14
rainy	takeit	14
rainy	leaveit	7

Table: Factor $f_5(F, U)$

Applying VEA: Step 3.2

Step 3.2: Find the optimal policy for Umbrella.

F	U	value
sunny	leaveit	49
cloudy	leaveit	14
rainy	takeit	14

Table: Finding the optimal policy for Umbrella from $f_5(F, U)$

F	U
sunny	leaveit
cloudy	leaveit
rainy	takeit

Table: Optimal policy for Umbrella

F	value
sunny	49
cloudy	14
rainy	14

Table: Factor $f_6(F)$ w/o Umbrella

Agent's Expected Utility of the Optimal Policy

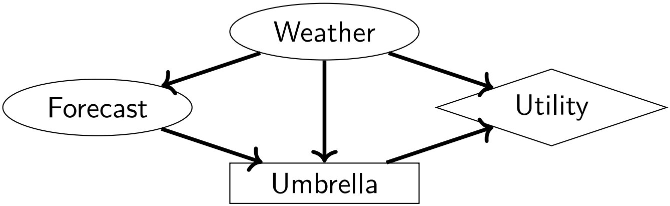
Sum out Forecast from $f_6(F)$ to produce $f_7()$.

value
77

Table: Factor $f_7()$

When following the optimal policy,
the agent's expected utility is 77.

What if I can observe the Weather directly?



Applying VEA: Step 1

Step 1: Remove all variables that are not ancestors of the utility node.

Every variable is an ancestor of the utility node.
There's nothing to be done.

Applying VEA: Step 2

Step 2: Define three factors. $f_1(W)$ for Weather, $f_2(W, F)$, and $f_3(W, U)$ for the Utility.

W	value
norain	0.7
rain	0.3

Table: Factor $f_1(W)$

W	F	value
norain	sunny	0.7
norain	cloudy	0.2
norain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

Table: Factor $f_2(W, F)$

Applying VEA: Step 2 (continued)

Step 2: Define three factors. $f_1(W)$ for Weather, $f_2(W, F)$, and $f_3(W, U)$ for the Utility.

W	U	value
norain	takeit	20
norain	leaveit	100
rain	takeit	70
rain	leaveit	0

Table: Factor $f_3(W, U)$

Applying VEA: Step 3.1

Step 3.1: Eliminate any variable that is not a parent of a decision node.

Weather and Forecast are both parents of Umbrella.

Nothing needs to be done for this step.

Applying VEA: Step 3.2

Step 3.2: Determine the optimal policy for Umbrella.

Find a factor that contains the decision node (Umbrella). All the other variables in the factor must be the decision node's parents.

$f_3(W, U)$ satisfies the requirements.

W	U	value
norain	takeit	20
norain	leaveit	100
rain	takeit	70
rain	leaveit	0

Table: Factor $f_3(W, U)$

Applying VEA: Steps 3.2

Step 3.2: Determine the optimal policy for Umbrella.

W	U	value
norain	leaveit	100
rain	takeit	70

Table: Finding the optimal policy for Umbrella

W	U
norain	leaveit
rain	takeit

Table: The optimal policy for Umbrella

W	value
norain	100
rain	70

Table: Factor $f_4(W)$ without Umbrella

Agent's Expected Utility Following the Optimal Policy

Determine agent's expected utility if they follow the optimal policy.

Eliminate Forecast.

$$\sum_F f_2(F, W) = f_5(W).$$

W	value
norain	1
rain	1

Table: Factor $f_5(W)$

Agent's Expected Utility Following the Optimal Policy

Determine agent's expected utility if they follow the optimal policy.

Eliminate Weather.
Multiply all the factors containing Weather.

$$f_1(W) \times f_4(W) \times f_5(W) = f_6(W).$$

Sum out Weather from $f_6(W)$ to produce $f_7()$.

$$\sum_W f_6(W) = f_7().$$

W	value
norain	70
rain	21

Table: Factor $f_6(W)$

value
91

Table: Factor $f_7()$

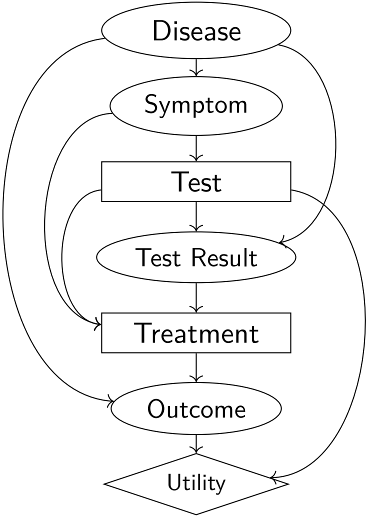
A Medical Diagnosis Scenario

Consider a simple medical diagnosis scenario. A doctor needs to make two decisions: choosing a test to perform and decides on a treatment based on the test results. The reason for performing a test is to obtain information (the test results) that may be useful for determining the treatment. It is often a good idea to test, even if testing itself may harm the patient.

The doctor can decide on which test to perform based on the patient's symptom. When deciding the treatment, the information available will be the patient's symptom, the tests performed, and the test results. The test results depend on the disease and what test was performed. The treatment outcome depends on the disease and the treatment performed. The patient's utility includes costs of tests and treatments, the pain and inconvenience to the patient in the short term, and the long-term prognosis.

(Example 9.12 in Poole and Mackworth)

Decision Network for the Diagnosis Scenario



VEA: Step 1

Step 1: Remove all variables that are not ancestors of the utility node.

All the variables are ancestors of the utility node.
There is nothing to be done for this step.

VEA: Step 2

Step 2: Define one factor for every random variable (chance node).

D	value
1	0.79
0	0.21

Table: Disease: $f_1(D)$

D	S	value
1	1	0.89
1	0	0.11
0	1	0.27
0	0	0.73

Table: Symptom: $f_2(D, S)$

VEA: Step 2

Define one factor for every random variable (chance node).

D	T	TR	value
1	1	1	0.22
1	1	0	0.78
1	0	1	0.55
1	0	0	0.45
0	1	1	0.91
0	1	0	0.09
0	0	1	0.53
0	0	0	0.47

Table: Test Result: $f_3(D, T, TR)$

D	Tr	O	value
1	1	1	0.59
1	1	0	0.41
1	0	1	0.16
1	0	0	0.84
0	1	1	0.93
0	1	0	0.07
0	0	1	0.65
0	0	0	0.35

Table: Outcome: $f_4(D, Tr, O)$

VEA: Step 2

Define one factor for the utility node.

T	O	value
1	1	900
1	0	600
0	1	1000
0	0	700

Table: Utility: $f_5(T, O)$

VEA: Step 3

There are still decision nodes remaining.
Eliminate every random variable that is not a parent of a decision node.

There are two such random variables: Disease and Outcome.

Let's eliminate Outcome first.

VEA: Step 3.1

Step 3.1: Eliminate Outcome.

Multiply all the factors that contain Outcome.

$$f_4(D, Tr, O) \times f_5(T, O)$$
$$= f_6(D, O, T, Tr)$$

Sum out Outcome.

$$\sum_O f_6(D, O, T, Tr) = f_7(D, T, Tr)$$

The new list of factors:
$f_1(D), f_2(D, S), f_3(D, T, TR), f_7(D, T, Tr)$.

VEA: Step 3.1 Factors

D	O	T	Tr	value
1	1	1	1	531
1	1	1	0	144
1	1	0	1	590
1	1	0	0	160
1	0	1	1	246
1	0	1	0	504
1	0	0	1	287
1	0	0	0	588
0	1	1	1	837
0	1	1	0	585
0	1	0	1	930
0	1	0	0	650
0	0	1	1	42
0	0	1	0	210
0	0	0	1	49
0	0	0	0	245

Table: Factor $f_6(D, O, T, Tr)$

D	T	Tr	value
1	1	1	777
1	1	0	648
1	0	1	877
1	0	0	748
0	1	1	879
0	1	0	795
0	0	1	979
0	0	0	895

Table: Factor $f_7(D, T, Tr)$

VEA: Step 3.1

Step 3.1: Eliminate Disease.

Multiply all the factors that contain Disease.

$$f_1(D) \times f_2(D,S) \times f_3(D,T,TR) \times f_7(D,T,Tr)$$
$$= f_8(D,S,T,TR,Tr)$$

Sum out Disease.

$$\sum_D f_8(D,S,T,TR,Tr) = f_9(S,T,TR,Tr)$$

The new list of factors: $f_9(S,T,TR,Tr)$.

VEA: Step 3.1 Factors

D	S	T	TR	Tr	value
1	1	1	1	1	120.2
1	1	1	1	0	100.2
1	1	1	0	1	426.1
1	1	1	0	0	355.4
1	1	0	1	1	339.1
1	1	0	1	0	289.3
1	1	0	0	1	277.5
1	1	0	0	0	236.7
1	0	1	1	1	14.85
1	0	1	1	0	12.39
1	0	1	0	1	52.67
1	0	1	0	0	43.92
1	0	0	1	1	41.92
1	0	0	1	0	35.75
1	0	0	0	1	34.30
1	0	0	0	0	29.25

Table: Factor $f_8(D, S, T, TR, Tr)$ first half

D	S	T	TR	Tr	value
0	1	1	1	1	45.35
0	1	1	1	0	41.02
0	1	1	0	1	4.486
0	1	1	0	0	4.057
0	1	0	1	1	29.42
0	1	0	1	0	26.90
0	1	0	0	1	26.09
0	1	0	0	0	23.85
0	0	1	1	1	122.6
0	0	1	1	0	110.9
0	0	1	0	1	12.13
0	0	1	0	0	10.97
0	0	0	1	1	79.54
0	0	0	1	0	72.72
0	0	0	0	1	70.54
0	0	0	0	0	64.49

Table: Factor $f_8(D, S, T, TR, Tr)$ second half

VEA: Step 3.1 Factors

S	T	TR	Tr	value
1	1	1	1	165.5
1	1	1	0	141.3
1	1	0	1	430.6
1	1	0	0	359.4
1	0	1	1	368.6
1	0	1	0	316.2
1	0	0	1	303.6
1	0	0	0	260.5

Table: Factor $f_9(S, T, TR, Tr)$ first half

S	T	TR	Tr	value
0	1	1	1	137.5
0	1	1	0	123.3
0	1	0	1	64.79
0	1	0	0	54.89
0	0	1	1	121.5
0	0	1	0	108.5
0	0	0	1	104.8
0	0	0	0	93.74

Table: Factor $f_9(S, T, TR, Tr)$ second half

VEA: Step 3.2

Step 3.2: Find the optimal policy for Treatment and eliminate it.

S	T	TR	Tr
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

Table: The optimal policy for Treatment

S	T	TR	value
1	1	1	165.5
1	1	0	430.6
1	0	1	368.6
1	0	0	303.6
0	1	1	137.5
0	1	0	64.79
0	0	1	121.5
0	0	0	104.8

Table: Factor $f_{10}(S, T, TR)$

The new list of factors: $f_{10}(S, T, TR)$

VEA: Step 3.1

There are still decision nodes remaining.
Eliminate every random variable that is not a parent of a decision node.

There is one such random variable: Test Result.

Let's eliminate Test Result.

VEA: Step 3.1

Step 3.1: Eliminate Test Result.
Multiply all the factors that contain Test Result.
Sum out Test Result.

$$\sum_{TR} f_{10}(S, T, TR) = f_{11}(S, T)$$

S	T	value
1	1	596.1
1	0	672.2
0	1	202.3
0	0	226.3

Table: Factor $f_{11}(S, T)$

The new list of factors: $f_{11}(S, T)$.

VEA: Step 3.2

Step 3.2: Find the optimal policy for Test and eliminate it.

S	T	value
1	0	672.2
0	0	226.3

Table: Finding the optimal policy for Test

S	T
1	0
0	0

Table: The optimal policy for Test

S	value
1	672.2
0	226.3

Table: Factor $f_{12}(S)$ without Test

The new list of factors: $f_{12}(S)$

Agent's Expected Utility following the Optimal Policy

Sum out all the remaining random variables.

Sum out Symptom.

$$\sum_{S} f_{12}(S) = f_{13}()$$

value
898.5

Table: Factor $f_{13}()$

When following the optimal policy,
the agent's expected utility is 898.5.

Revisiting the Learning Goals

By the end of the lecture, you should be able to

- Given a decision network with a single decision, determine the optimal policy and the expected utility of the optimal policy by enumerating all the policies.
- Given a decision network with a single decision, determine the optimal policy and the expected utility of the optimal policy by applying the variable elimination algorithm.
- Given a decision network with sequential decisions, determine the optimal policy and the expected utility of the optimal policy by applying the variable elimination algorithm.