Decisions with Multiple Agents: Game Theory

Alice Gao

Lecture 17

Based on work by K. Leyton-Brown, K. Larson, and P. van Beek

Outline

Learning Goals

By the end of the lecture, you should be able to

- Determine dominant-strategy equilibria of a 2-player normal form game.
- Determine pure-strategy Nash equilibria of a 2-player normal form game.
- Determine Pareto optimal outcomes of a 2-player normal form game.
- Calculate a mixed strategy Nash equilibrium of a 2-player normal form game.

Prisoner's dilemma

		Anna: refuse	Anna: testify
Alice	refuse	$(-1, -1)$	$(-3, 0)$
	testify	$(0, -3)$	$(-2, -2)$

CQ: Prisoner's dilemma - dominant strategy equilibrium

CQ: Does this game have a dominant strategy equilibrium?
If so, which outcome is such an equilibrium?

(A) (refuse, refuse)

(B) (refuse, testify)

(C) (testify, refuse)

(D) (testify, testify)

(E) There is no dominant strategy equilibrium.

		Anna	
		refuse	testify
Alice	refuse	$(-1, -1)$	$(-3, 0)$
	testify	$(0, -3)$	$(-2, -2)$

CQ: Prisoner's dilemma - Nash equilibria

CQ: How many of the four outcomes are **pure strategy Nash equilibria**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

		Anna: refuse	Anna: testify
Alice	refuse	$(-1, -1)$	$(-3, 0)$
	testify	$(0, -3)$	$(-2, -2)$

CQ: Prisoner's dilemma - Pareto optimality

CQ: How many of the four outcomes are **Pareto optimal**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

		Anna: refuse	Anna: testify
Alice	refuse	$(-1, -1)$	$(-3, 0)$
	testify	$(0, -3)$	$(-2, -2)$

Matching quarters

		Anna: heads	Anna: tails
Alice	heads	$(1, 0)$	$(0, 1)$
	tails	$(0, 1)$	$(1, 0)$

Alice wants the two coins to match whereas Anna wants the two coins to mismatch.

CQ: Matching quarters - Nash equilibria

CQ: How many of the four outcomes are **pure strategy Nash equilibria**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

		Anna: heads	Anna: tails
Alice	heads	$(1, 0)$	$(0, 1)$
Alice	tails	$(0, 1)$	$(1, 0)$

Conflicting interests

		Anna	
		dancing	*concert*
Alice	*dancing*	$(2, 1)$	$(0, 0)$
	concert	$(0, 0)$	$(1, 2)$

Alice and Anna want to sign up for an activity together.
They both prefer to sign up for the same activity.
However, Alice prefers dancing over a concert whereas Anna prefers a concert over dancing.

CQ: Conflicting interests - mixed strategy Nash equilibria

CQ: At the mixed strategy Nash equilibrium,
with what probability does **Alice go dancing**?

(A) [0, 0.2) (B) [0.2, 0.4) (C) [0.4, 0.6) (D) [0.6, 0.8)
(E) [0.8, 1]

		Anna	
		dancing	*concert*
Alice	*dancing*	$(2, 1)$	$(0, 0)$
	concert	$(0, 0)$	$(1, 2)$

CQ: Conflicting interests - mixed strategy Nash equilibria

CQ: At the mixed strategy Nash equilibrium,
with what probability does **Anna go dancing**?

(A) [0, 0.2) (B) [0.2, 0.4) (C) [0.4, 0.6) (D) [0.6, 0.8)
(E) [0.8, 1]

		Anna	
		dancing	*concert*
Alice	*dancing*	$(2, 1)$	$(0, 0)$
	concert	$(0, 0)$	$(1, 2)$

Revisiting the Learning Goals

By the end of the lecture, you should be able to

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- Determine pure-strategy Nash equilibria of a 2-player normal form game.
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- Calculate a mixed strategy Nash equilibrium of a 2-player normal form game.